

**STUDENT VERSION
 SPREAD OF INFORMATION**

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STATEMENT

Approximately 4% of 18-24 year-olds believe the Earth is flat.¹ Flat-Earthers spread additional supportive theories and those theories permeate throughout the community² in a way that can be modeled mathematically. Misinformation, disinformation, and correct information all tend to permeate similarly within any closed population of individuals open to the new information. This exercise explores the spread of information using coins to model the proportion of the population who ‘know’.

Part 1: An experiment to model the spread of information


- 1) Begin with 20 coins in a row, tails up. Turn one coin heads up.
 - Heads represent those who know.
 - Tails represent those who do not know.
- 2) Read through steps A-D. You will repeat steps A-D several times. Once you understand the experiment (a sample is provided), guess what the final two graphs from Step 4 will look like and sketch your predictions on the two graphs.
 - A) For each iteration – Reorder the coins randomly (without flipping any) and put them in a horizontal line then complete a row in the table. Completed example of iteration 0 shown:

Iteration	How many know	Proportion who know	How many don't know	Increase in how many know
0	1	1/20	19	1

- B) Those who ‘know’ spread the information to a proportional amount of the population. So, beginning with the left-most coin, move to the left the proportion who are told. For the first iteration, this would be one coin. See examples.
- C) Some of the population on the left will already know, some will not. Flip any who “do not know” (tails) to become “know” (heads).

Completed example of iteration 2 shown.

Iteration	How many know	Proportion who know	How many don't know	Increase in how many know
0	1	1/20	19	1
1	2	2/20	18	1
2	3	3/20	17	1

Note: This entry is 1, not 2, because two were told, but one already knew 

- D) Repeat steps A-D until all know.

- 3) Before completing the experiment (perhaps after you have finished a few iterations), sketch a quick predictive graph of the number who know and also a quick predictive graph of the increase in how many know.

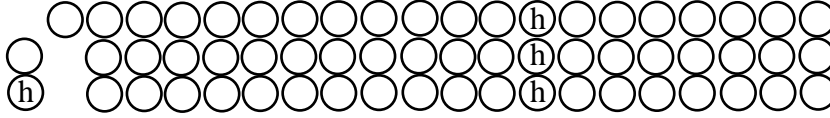
Sample Iterations

Begin: One knows, the rest do not



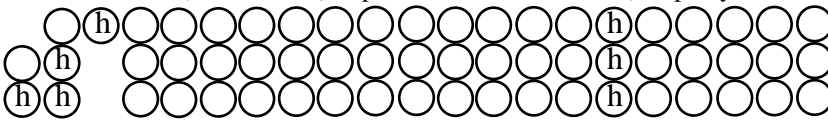
#	know	k/20	don't	+
0	1	1/20	19	1

Iteration 1: A) Reorder; B) Separate those to be told; C) Flip any “don’t knows” to “know”



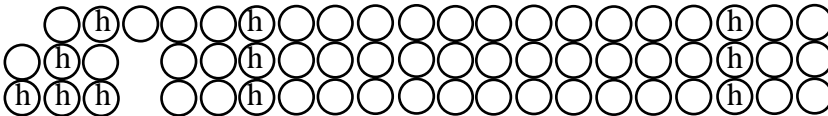
#	know	k/20	don't	+
0	1	1/20	19	1
1	2	2/20	18	1

Iteration 2: A) Reorder; B) Separate those to be told; C) Flip any “don’t knows” to “know”



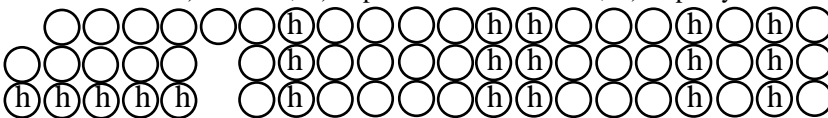
#	know	k/20	don't	+
0	1	1/20	19	1
1	2	2/20	18	1
2	3	3/20	17	1

Iteration 3: A) Reorder; B) Separate those to be told; C) Flip any “don’t knows” to “know”



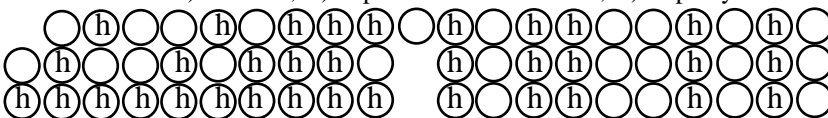
#	know	k/20	don't	+
0	1	1/20	19	1
1	2	2/20	18	1
2	3	3/20	17	1
3	5	5/20	15	2

Iteration 4: A) Reorder; B) Separate those to be told; C) Flip any “don’t knows” to “know”



#	know	k/20	don't	+
0	1	1/20	19	1
1	2	2/20	18	1
2	3	3/20	17	1
3	5	5/20	15	2
4	10	10/20	10	5

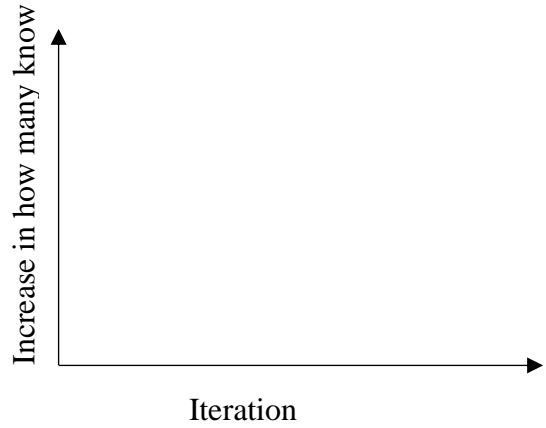
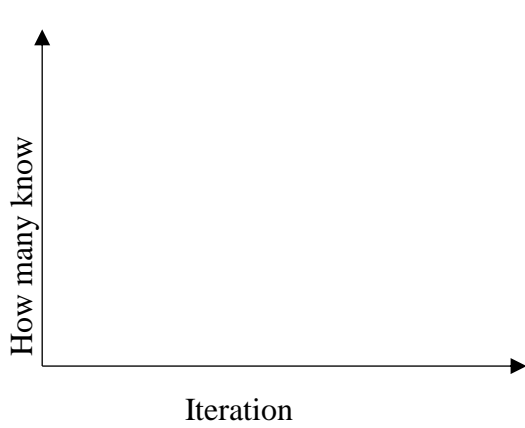
Iteration 5: A) Reorder; B) Separate those to be told; C) Flip any “don’t knows” to “know”



#	know	k/20	don't	+
0	1	1/20	19	1
1	2	2/20	18	1
2	3	3/20	17	1
3	5	5/20	15	2
4	10	10/20	10	5
5	15	15/20	5	5

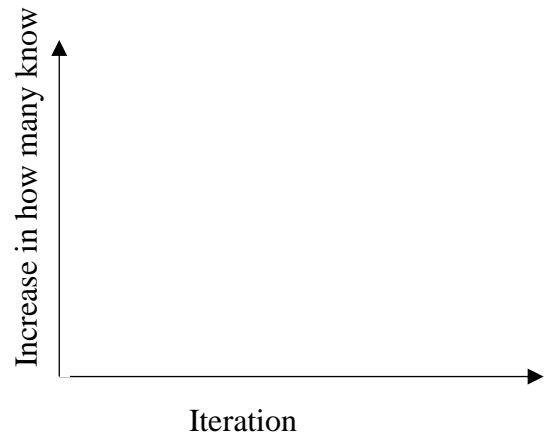
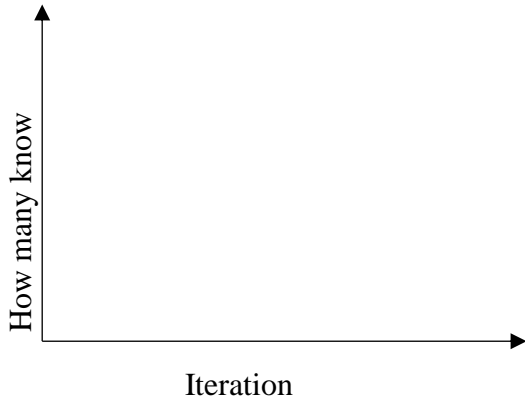
Etc.

Before completing the experiment, sketch a quick predictive graph of the number who know and also a quick predictive graph of the increase in how many know. Consider concavity. Note/hint: one should be the antiderivative of the other.



Iteration	How many know	Proportion who know	How many don't know	Increase in how many know
0	1	1/20	19	1
1				
2				
3				
4				
5				
6				
7				

4) Graph your data below.



5) Compare your data graphs to your predictions. What features did you predict correctly? What features were a surprise? Was your concavity correct?

Part 2: Attempt at a mathematical model

Part 1 was an experiment with population size 20. Let us attempt to build a general mathematical model of this scenario, using population size M , and using $a(n)$ to represent the number of heads in the population at iteration n .

- 1) What would $a(0)$ be in this scenario?

$$a(0) =$$

- 2) Each next iteration should arise based on how many already 'know,' plus how many additional people are added. Consider the following:

$$a(1) = \textit{those who already know} + \textit{those who are told but don't already know}$$

$$a(1) =$$

- 3) Complete the expression for $a(2)$ then write the general case for $a(n+1)$ in terms of $a(n)$:

$$a(2) =$$

$$a(n + 1) =$$

- 4) Check that your result is equivalent to the answer: $a(n + 1) = 2a(n) - \frac{a(n)^2}{M}$

- 5) Examine the validity of this equation by calculating $a(1)$ and $a(2)$. Do they match your answers from above?

- 6) Examine the validity of the above equation by calculating $a(n+1)$ if $a(n)=M$. Is this result reasonable? Why or why not?

- 7) Examine the validity of the above equation by calculating $a(1)$ if $a(0)=0$. Is this result reasonable? Why or why not?

Part 3 (Challenge): Another approach -- modeling change as a function

An iterative model can be useful, but a function model would be more useful. To find a function, $b(t)$ using $a(n)$, we will create a derivative and apply differential equation techniques to find $b(t)$, the mathematical model we seek.

1) Begin by using the answer from above in the form:

$$a(n + 1) = a(n) + \left(a(n) - \frac{a(n)^2}{M} \right) \Delta n, \text{ (where } \Delta n = 1) \text{ to write a function, } b(t + \Delta t).$$

$$b(t + \Delta t) =$$

2) Solve for $\frac{b(t + \Delta t) - b(t)}{\Delta t}$

$$\frac{b(t + \Delta t) - b(t)}{\Delta t} =$$

3) Use a limit to arrive at b' which is separable.

$$b' =$$

Part 4: Solving the separable differential equation

1) Solve the above for $b(t)$ giving $\frac{A \cdot M}{1 + A e^{t/M}}$ where M is the population size, and A is determined by an initial value.

$$b' =$$

$$b(t) =$$

2) Compare this equation with your data by first determining A , then comparing $b(t)$ to your data using either a table, a graph or both.

¹ <https://www.livescience.com/62220-millennials-flat-earth-belief.html>

² <https://www.netflix.com/title/81015076>