Problem A: Group Affinity and Fashion Sense

People tend to congregate into groups in different ways. People can create strong links in small cliques or identify loosely as part of a larger trend. One example of the latter phenomena is hipsters. An amusing example of how someone identified and adapted their appearance to conform to the look of a stereotypical hipster is a person who complained that his image was used in an article about people conforming to a stereotypical look [1]. It later turned out the image in the article [2] was someone else who happened to look like the person complaining that image was appropriated. This raises a number of questions about how people choose which groups to associate with as well as how they decide to adjust to the expectations of the other people within their group. We focus on the latter question, and you are asked to examine the propensity for a person to alter their appearance and conform to particular expectations. You should develop a model that describes how different people within an established group interact and decide to change some particular part of their appearance. How long does it take for people in the group to change their appearance? How many people will change, and how much alike will they eventually appear? You should provide an analysis about which parts of your model impact how different subgroups change and how quickly the change occurs.

Your description of the model should include the following:

- Clearly describe the aspect that is being changed.
- Describe how information about it is exchanged between people in the group.
- Describe the way people interact and how the model mimics those interactions.
- Describe the range of values of the parameters and the meaning associated with higher versus lower values of these parameters.

Note, the work that sparked this exchange included a mathematical model of how people decide whether or not to conform and change their appearance [3]. The model and analysis in the paper is advanced, and it is not a good starting point for the beginning development of a model. The model in the paper only included two different groups: conformists and nonconformists. It also included a delay to approximate the interactions between the two groups. (A delay can be difficult to approximate and analyze and should probably be avoided for a first effort in a short term project.)

References


Problem B: Movement Of An Object In Microgravity Environments

In February 2019 a Japanese probe made contact with a small asteroid, Ryugu [1]. The team overseeing the program had to overcome a number of technical challenges. For this question we focus on the issues associated with a low gravity environment. The team had to land a probe gently enough so that it does not bounce and move too far away from a designated landing position. The next problem is moving the probe to a new position using a minimal amount of energy and also minimizing how far the probe bounces on the surface of the asteroid.

You have been asked to provide guidance in helping find a new asteroid on which to land a probe. The goal is to determine the range of dimensions for the smallest possible asteroids which can be used to land a probe. (Keep in mind that asteroids can have high aspect ratios and are generally not round.) Your team should develop a method to land a small probe on the asteroid and the final position of the probe after coming to rest should be as close as possible to a predetermined landing point. At the same time the amount of bouncing should be as small as possible to avoid damaging the probe. You should also develop a way to move the probe to a predetermined position using a spring that will allow the probe to hop in a given direction without using a device that generates thrust. The analysis you provide should include a detailed description of the mathematical models you develop to describe the movement of the probe in these different conditions.

The surface of the asteroid is assumed to be quite rugged, and the probe may have to jump into a ravine or along the side of a steep cliff. You should provide guidance concerning the limits of moving the probe using a minimal number of jumps under a wide variety of situations. Your analysis should include a description of the possible limits to what area can be explored and the description should include guidance on choosing an asteroid with respect to the possible dimensions.

References
SCUDEM IV 2019

Problem C: Chemical Espionage

It can be difficult for some insects to find mates. One common way for a female to attract a male is to use chemical signals. One problem with this approach is that this signaling can attract many males, and in response the males often use chemical signals, called anti-aphrodisiacs, that are used to either mask or dissuade other males.

An example of this can be found in the large cabbage white butterfly *Pieris brassicae*. Unfortunately for the butterflies, the chemical signals can be exploited by parasitic wasps. Two species of wasps have been identified that can detect the anti-aphrodisiacs, and when a female butterfly has the chemical signal the wasps are more likely to follow the butterfly and lay their own eggs in the butterflies’ eggs.

These interactions introduce two competing pressures on the butterfly population. For the male butterflies the anti-aphrodisiacs make it more likely for them to fertilize eggs. For the female butterflies the anti-aphrodisiacs make it less likely to be bothered by more males, and the females can focus on placing their eggs in the most advantageous place. On the other hand the anti-aphrodisiacs make it more likely these eggs will be eaten by the wasp larvae.

One question that arises is to determine the trade-offs and balance between the two competing interests. To do so develop a mathematical model for the interactions of the male and female *P. brassicae* as well as the parasitic wasps. What is the best balance for this system and what is likely to happen in the long run?

References

You are to select one of the posed Additional Issues for your model and address it in your Presentation. You are NOT to redo your model. Rather you are to describe how you would implement the issue in your model and indicate what you believe would be the effects of the additional issue. In your presentation be sure to state when you are discussing the Additional Issue.

**Problem A:**

1) A company that makes the items associated with the aspect you included in your model wishes to manipulate people in order to increase sales. What should they do with respect to changing the item or changing their advertising that will result in the greatest sales increase?

2) The original statement asks you to model one aspect that is adopted within a group. How would your model change if you are required to model how two aspects are adopted within a group?

3) How does your model depend on the initial condition? For example, will it always behave the same or is it possible to determine an initial condition that will yield a different end behaviour? What does your answer to this question indicate about the model?

**Problem B:**

1) If your model assumes that the probe cannot roll when it lands on the surface how would you change the model if it can roll? Likewise, if your model assumes it can roll how would you change the model if it cannot roll? How do you adapt your model for various propensities to roll?

2) What role does the size and shape of the asteroid play in your model? If the size of the asteroid changes what impact does that have on your predictions?

3) You are asked to determine a way for you to make a probe travel around the circumference of an asteroid after it has landed. How can you adapt your model to determine the best way to move along a given path and stay as close as possible to the path?

**Problem C:**

1) Suppose you are asked to add an animal that is a predator of both the butterflies and the wasps, a bird for example. How would you change your model to accommodate this new situation?

2) Suppose that the female butterfly could detect a male butterfly’s propensity to use the anti-aphrodisiac prior to mating. Based on your model’s results what should her strategy be in choosing a mate?

3) How would your model change if the effectiveness of the anti-aphrodisiac depends on the time of the day? For example, the anti-aphrodisiac may not be as effective in lower temperatures in the early morning as opposed to midday.