Simiode - Problem C
Chemical Espionage
SCUDEM IV 2019

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Problem Overview

- Large cabbage white butterfly (LCWB) - *Pieris Brassicae*
- Chemical signaling
  - Aphrodisiac (A)
  - Anti-aphrodisiac (AA)
- 2 parasitic wasp species
- Interests
  - Butterfly population increase
  - Wasp population increase

Variables

- $A_o =$ initial butterfly population
- $A_i =$ population after i^{th} mating season
- $L =$ number of eggs laid by a LCWB during the mating season (RV)
- $\bar{L} =$ average number of eggs laid by a LCWB during the mating season
- $W =$ number of eggs parasitized per clutch (RV)
- $\bar{W} =$ average number of eggs parasitized per clutch
- $p =$ probability that mated female is mounted by wasp
- $d =$ death rate per mating season for LCWB
- $\bar{L} - p\bar{W} =$ expected number of surviving LCWB eggs per clutch

Research & Assumptions

- Research
  - $20 \leq L \leq 50$ laid eggs per mating season for each female butterfly
  - $20 \leq W \leq 50$ eggs can be parasitized once wasps attaches itself
  - $2 - 3$ mating seasons per year
- Assumptions
  - Both wasp species behave the same
  - Half the population at any time is female
  - Female butterflies remain monogamous per mating season, i.e., females only give birth to one clutch per season
  - $\bar{W} \leq \bar{L}$
  - p remains constant as the butterfly population fluctuates
    - More accurately, p is a function of the wasp population
  - Every female reproduces during a mating season

Discrete Model

Generation	Population
0	A_0
1	$(A_0 - A_0 d) + \frac{1}{2}(A_0 - A_0 d)(\bar{L} - p\bar{W})$ $A_0(1-d)\left(1 + \frac{1}{2}(\bar{L} - p\bar{W})\right)$
2	$A_1(1-d)\left(1 + \frac{1}{2}(\bar{L} - p\bar{W})\right)$ $A_0(1-d)\left(1 + \frac{1}{2}(\bar{L} - p\bar{W})\right)(1-d)\left(1 + \frac{1}{2}(\bar{L} - p\bar{W})\right)$ $A_0(1-d)^2\left(1 + \frac{1}{2}(\bar{L} - p\bar{W})\right)^2$
...	...
n	$A_0(1-d)^n\left(1 + \frac{1}{2}(\bar{L} - p\bar{W})\right)^n$

Visual Model

x	$A(x)$
0	2
1	5.125
2	13.132813
3	33.652832
4	86.235382
5	220.97817
6	566.25655
7	1451.0324
8	3718.2706
9	9528.0683
10	24415.675

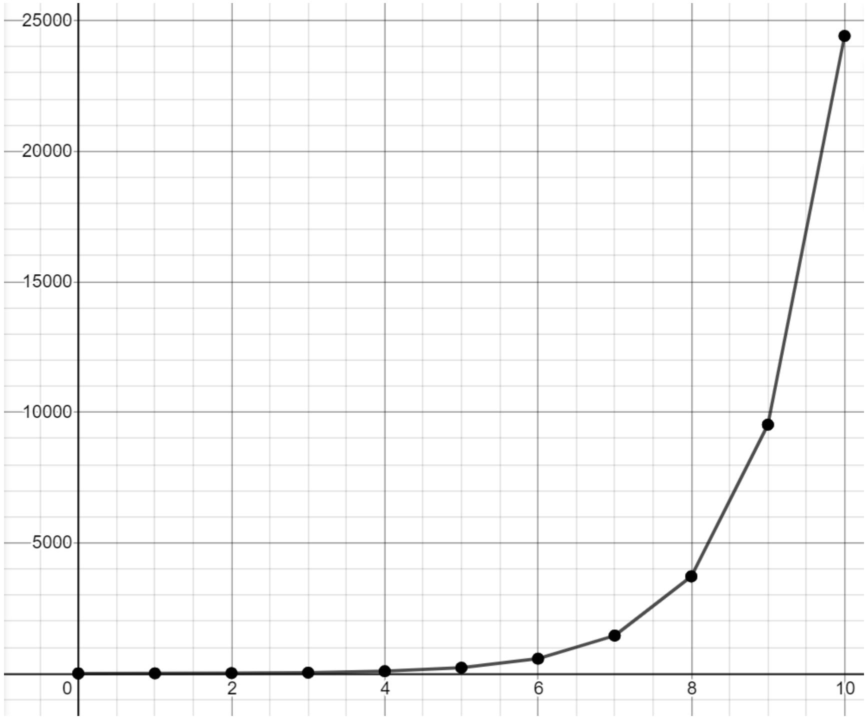


Table 1: Visualization of discrete population model

Parameters:

$A_0 = 2$

$p = 0.5$

$d = 0.75$

$\bar{L} = 35$

$\bar{W} = 33$

Horizontal axis: generation number

Vertical axis: butterfly population size

Drawbacks of Model

- Carrying capacity not factored into equation
  - Population increases without bounds
- Parameters likely difficult to experimentally obtain
  - Rare species of butterfly
  - Varying scientific conclusions about population parameters

Factoring Carrying Capacity

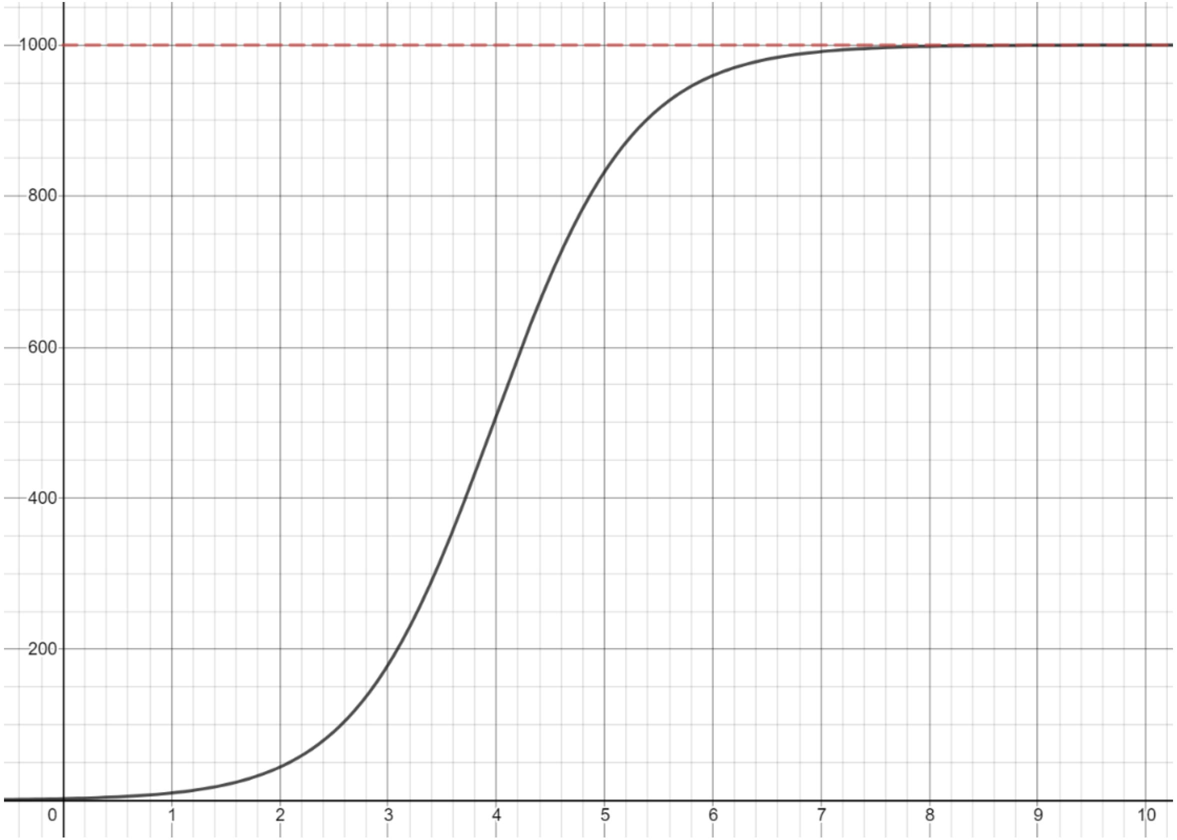
- $A_{i+1} - A_i = rA_i$ = initial model
- $r = -d + \frac{1-d}{2}(\bar{L} - p\overline{W}) =$ rate of increase of population size
- $K =$ carrying capacity
- $\frac{A_i}{K} =$ fraction of carrying capacity used
- $1 - \frac{A_i}{K} =$ fraction of carrying capacity not used
- $A_{i+1} - A_i = rA_i\left(1 - \frac{A_i}{K}\right) =$ model set proportional to unused fraction
- $A_{i+1} - A_i = (-d + \frac{1-d}{2}(\bar{L} - p\overline{W}))A_i(1 - \frac{A_i}{K})$

From this recurrence relation we get the following continuous differential equation.

Continuous Model

- $\frac{dA}{dt} = rA\left(1 - \frac{A}{K}\right) \rightarrow \int \frac{dA}{A(\frac{1-A}{K})} = \int rdt$
- $\int \left(\frac{1}{A} + \frac{1}{K-A}\right) dA = \int rdt \rightarrow \ln|A| - \ln|K-A| = rt + C$
- $ln\left|\frac{K-A}{A}\right| = -rt - c \rightarrow \left|\frac{K-A}{A}\right| = e^{-rt-c} \rightarrow \frac{K-A}{A} = C_1 e^{-rt}$
- $A = \frac{K}{1+Ce^{-rt}}$ general solution
    - $C = \frac{K-A_o}{A_o}$
- $A = \frac{K}{1+\frac{(K-A_o)}{A_o}e^{-rt}}$ exact solution

Visual Continuous Model



Horizontal axis: generation number
Vertical axis: butterfly population size

Parameters:
$A_0 = 2$
$p = 0.5$
$d = 0.75$
$\bar{L} = 35$
$\bar{W} = 33$
$K = 1000$

Drawbacks of Continuous Model

- Unrepresentative of natural situation
    - Mating season only occurs 2-3 times per year
- Parameters difficult to experimentally obtain
    - Rare species of butterfly
    - Varying scientific conclusions about population

The Long Run

- Discontinuous model
  - Butterfly population continues to increase exponentially without bound
- Continuous model
  - Butterfly population will increase exponentially until carrying capacity begins limiting the growth rate

Conclusion

- Population dynamics of butterfly population and wasp population can be modeled in a discrete and continuous setting quite easily
- The discrete model with the added carrying capacity factor takes into account the finite nature of mating seasons
- The continuous model yielded a solution that only depended upon time
- Lack of concrete data hinders the accuracy and real-world application of these models
- As for the relationship between the male and female butterflies, we failed in finding any meaningful models
- Wasp population was not modeled for simplification purposes; taking this into account makes p a function of the wasp population

References

Huigens, M. E., Woelke, J. B., Pashalidou, F. G., Bukovinszky, T., Smid, H. M., & Fatouros, N.
E. (2010). Chemical espionage on species-specific butterfly anti-aphrodisiacs by hitchhiking Trichogramma wasps. *Behavioral Ecology*, *21*(3), 470–478. doi: 10.1093/beheco/arq007

Cabbage White Butterfly. (n.d.). Retrieved November 9, 2019, from https://www.animalspot.net/cabbage-white-butterfly.html.