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Problem C

The cabbage white butterfly *Pieris brassicae (P. brassicae)*, uses chemical signals to facilitate reproduction. Unfortunately for the butterflies, the chemical signals can be exploited by parasitic wasps. Two species of wasps have been identified that can detect the chemical signature of mated female *P. brassicae*. The wasps tend to follow the signal and lay their own eggs in the butterflies eggs.

These interactions produce two competing pressures on the butterfly population. For the butterflies, the chemical signals enhance reproduction; however, these signals make it more likely these eggs will be eaten by wasp larvae.

The goal of this project is to determine the trade-offs and balance between the two competing interests by developing a mathematical model for the interactions of the female and male *P. brassicae* and parasitic wasps.

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Initial Assumptions

(1) Sex Ratio of offspring is 1 to 1 for wasps and butterflies
(2) Mortality rate for male and female *P. brassicae* is identical
(3) Mortality rate for male and female wasps is identical
(4) Differences between *T. evanescens* and *T. brassicae* are negligible enough that that they can be combined and averaged for the purposes of their parasitic effects on *P. brassicae*
(5) Under field conditions, T. evanescens and T. brassicae respond to mated P. Brassicae in the same way the respond to P. Rapae females under lab conditions.
(6) Adult mortality rate for *T. evanescens* and *T. brassicae* is 3.45% per day
(7) Adult mortality rate for *P. brassicae* is 6.75% per day
(8) Female wasps lay eggs in batches of approximately 35

More Assumptions

(9) No other parasites are known to exist in the modeled environment that would affect offspring

(10) Initial male population is at least 75% of female population in both species

(11) P. brassicae lay eggs in batches of 50-100.

Constants and Variables

(1) Bx = *P. brassicae* females
(2) By = *P. brassicae* Male
(3) Wx = Female Wasps
(4) Wy = Male Wasps
(5) $\underline{\alpha}$ = *P. brassicae* Clutch Size
(6) $\mathcal{B}$ = Wasp Success Rate
(7) Δ = Adult mortality rate for *P. brassicae*
(8) θ = Wasp Clutch Size
(9) Ω = Adult mortality rate for wasps

Not to Scale

System of equations

Although there are various approaches to reproductive models, we chose to represent the reproductive interaction between male and female in the form of $\sqrt{Bx + By}$ and $\sqrt{Wx + Wy}$ in order to keep our model simple since we are assuming the ratio of male to female offspring with remain one-to-one.

We chose to operate the model with two equations, one for the change in butterfly population and another for the change in wasp population:

$$\frac{dB}{dt} = (Bx + By) + \alpha \frac{\sqrt{Bx+By}}{\beta\sqrt{Wx+Wy}} - \triangle(Bx + By)$$

$$\frac{dW}{dt} = (Wx + Wy) + \beta\theta \frac{\sqrt{Bx+By}}{\sqrt{Wx+Wy}} - \Omega(Wx + Wy)$$

System of equations

In our approximation, we chose to imbed separate equations of the following form for the male and female *P. brassicae* and for the male and female wasps in order to separately approximate the change in the respective male and female populations:

$$\frac{dBx}{dt} = Bx + \left(\frac{\alpha}{2} \cdot \frac{\sqrt{Bx+By}}{\beta\sqrt{Wx+Wy}}\right) - \triangle(Bx)$$

$$\frac{dBy}{dt} = By + \left(\frac{\alpha}{2} \cdot \frac{\sqrt{Bx+By}}{\beta\sqrt{Wx+Wy}}\right) - \triangle(By)$$

$$\frac{dWx}{dt} = Wx + \left(\beta \cdot \frac{\theta}{2} \cdot \frac{\sqrt{Bx+By}}{\sqrt{Wx+Wy}}\right) - \Omega(Wx)$$

$$\frac{dWy}{dt} = Wy + \left(\beta \cdot \frac{\theta}{2} \cdot \frac{\sqrt{Bx+By}}{\sqrt{Wx+Wy}}\right) - \Omega(Wy)$$

As you might expect with the sex ratios of the offspring assumed to be one-to-one, the male-female balance of both species remained symmetrical. However, we expect this model would predict an unreasonably a large rate of growth if there were a large excess of the more common sex.

Additional Issues

We were presented with the problem of exploring the best balance for this ecosystem and what is likely to happen in the long run.

If our assumptions hold, our model predicts that the populations of the two species will adjust to equilibrium within a fairly short time and remain at equilibrium.

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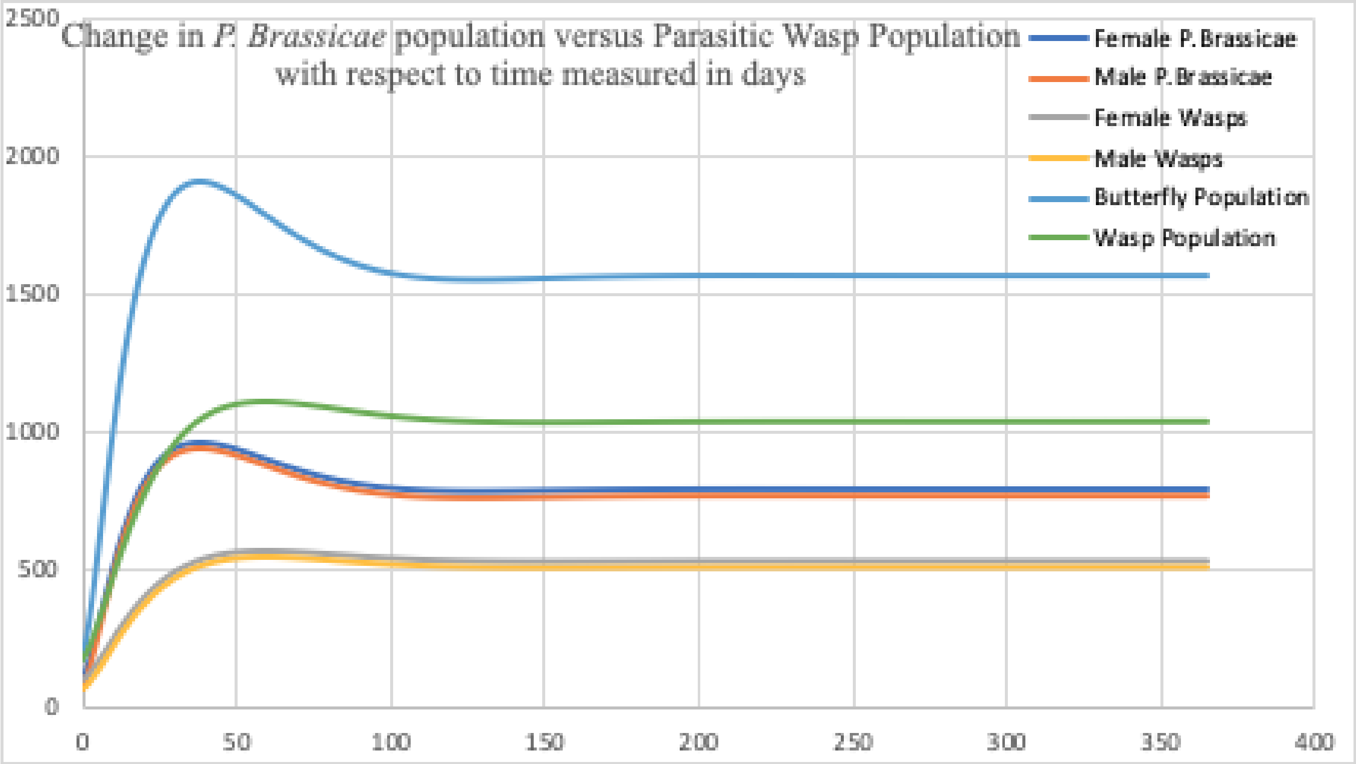
Graphs/Results



Conclusion

We experimented with varying the birth rates and mortality rates of the individual species; however, our model suggests that the two species adjust to an equilibrium state in fairly short order and remain at equilibrium. Our model produces a reasonable conclusion as long as our assumptions hold. However, there are other ways to model the reproductive behavior of the two species such that effects favoring the survivability of one sex in either or both species can be more accurately predicted. Our model, although less mathematically complex, would produce unreasonable growth rates based on the more common sex.

In the end this was a challenging problem that was very interesting to tackle and was a fun experience.

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References

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