A Mathematical Model of Chemical Espionage

Problem choice: Chemical Espionage
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Additional Problem C
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Our Problem

- The two different competing pressures on the female butterflies
- Males release Anti-Aphrodisiacs
- Wasps are attracted to the released chemical
- This signal makes it harder to have eggs

When parents ask for computer help

Continued

- The Process:
  - Mating between butterflies occurs
  - Males release signal (anti-Aphrodisiacs)
  - Wasps are attracted to the signal released by Males while females can better plant eggs
  - Wasps Follow Females
  - Wasps Lay eggs
  - Wasp Eggs eat butterfly eggs

Building the Model

- We can neglect Wasps Mounting virgin females, only followed mated Females
- There are 4 stages in a butterflies life cycle but we only assumed from Egg to Adult
- Constant reproduction rate for wasps
- Eggs for Male and Female are 50/50
- 50% adults female/male
- Wasps 15-20 days as an egg
- Environment is always the same
- Assumes adequate access brassica plants for breeding, consistent breed (i.e. cauliflower)
- Females monogamous (Fm is a terminal state)
- Adults live 3-12 days
- Only 1 Wasp population (there are multiple species)
- When male and females come in contact automatically results in some number of eggs
- All the eggs that survive the wasps become either adult males or females
- Cyclic, timed behavior b/c season-sensitive, mating window small so relatively in-sync

What We expected

- We expected cyclic behaviour
- Behavior without Wasps: Males and Mated Females reach a stability but Eggs and Virgin Females should be unstable
  - Females and males mate-creating an incline in eggs
  - Eggs all become adults
  - Only issue is when males mate with females less males are inclined to mate with the female
- Behavior with Wasps: Nothing should be approaching a stabile point
  - Difference is that Wasps are now killing Butterfly Eggs

Our Model (Differential Equations)

$$\frac{dE}{dt} = bd_{Fm}F_m - (\omega + \sigma)F_m W - d_E E \quad (1)$$

$$\frac{dFv}{dt} = \lambda E - (\alpha + \sigma)F_v M - d_{Fv}F_v \quad (2)$$

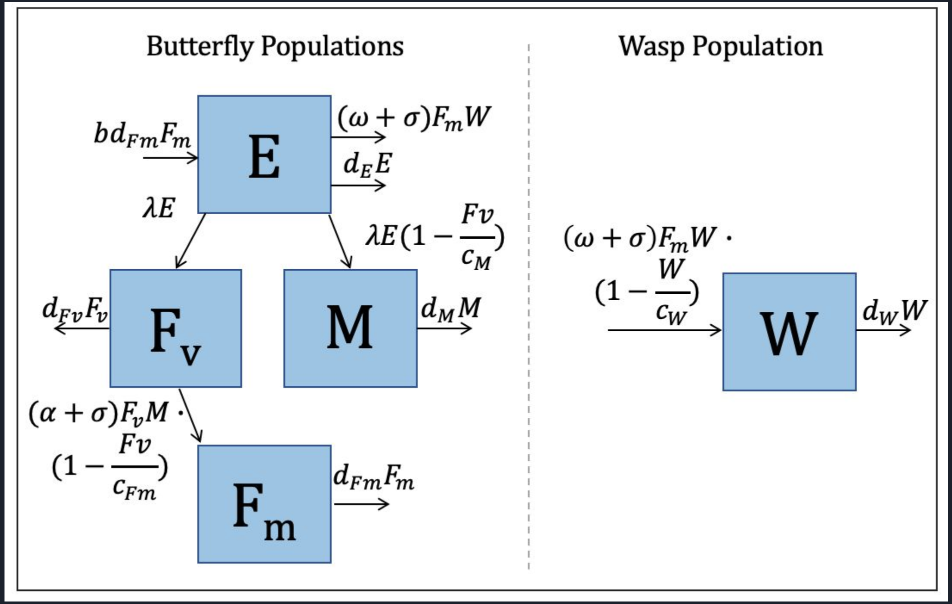
$$\frac{dM}{dt} = \lambda E(1 - \frac{F_v}{c_m}) - d_M M \quad (3)$$

$$\frac{dFm}{dt} = (\alpha + \sigma)F_v M(1 - \frac{F_v}{c_{Fm}}) - d_{Fm}F_m \quad (4)$$

$$\frac{dW}{dt} = (\omega + \sigma)F_m W(1 - \frac{W}{c_w}) - d_W W \quad (5)$$

$$N_{Adult\ butterflies} = Fv + Fm + M$$

Our Model (Diagram)



Parameters:

b: birth rate

ƛ: transition rate from eggs to adults

dfv: death rate of virgin female butterflies

dm: death rate of male butterflies

dw: death rate of wasps

dfm: enhanced death rate of mated females

de: death rate of eggs

ω: rate of interaction between mated females and wasps

σ: amount of anti-aphrodisiac

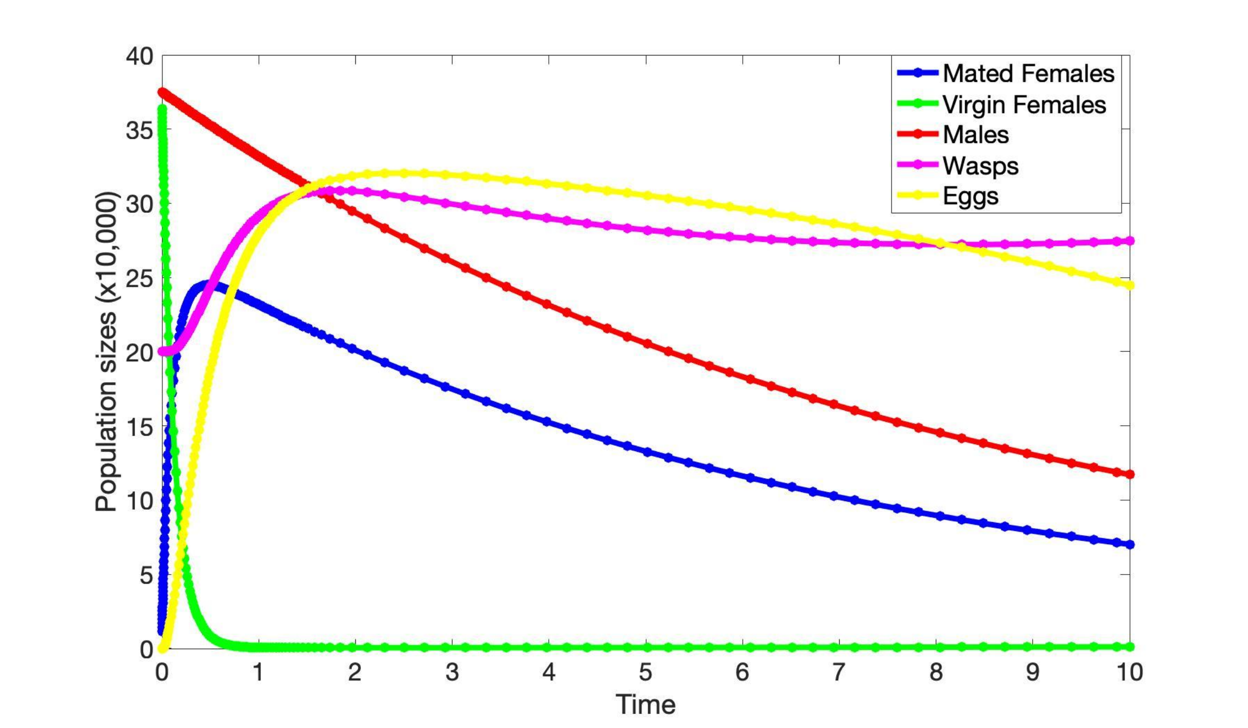
α: rate of interaction between virgin females and males

cfm: carrying capacity of mated females

cm: carrying capacity of males

cw: carrying capacity of cw

Results

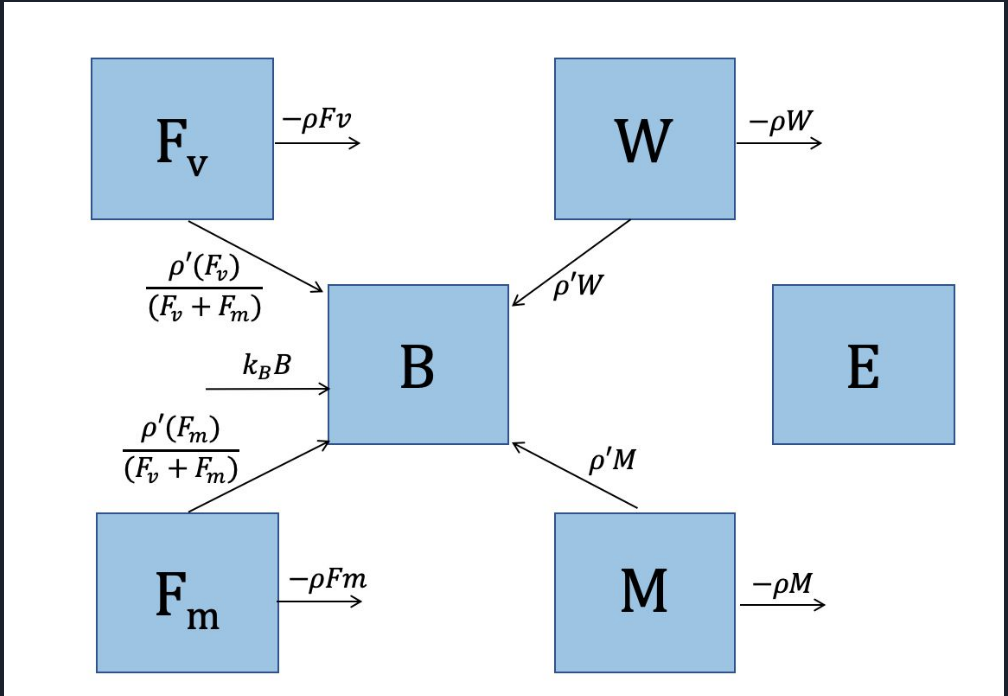


Mated Females
Virgin Females
Males
Wasps
Eggs
Population sizes (x10,000)
Time
0
5
10
15
20
25
30
35
40
0
1
2
3
4
5
6
7
8
9
10

Additional Issue

- Introduction of a predator of the butterflies and wasps, suppose an insect-eating bird (i.e. *Cardinalis cardinalis* (cardinal)), *B*
- Assume bird eats all insects indiscriminately at same rate, ρ (proportion of insect populations eaten by B per day) so multiplied by each adult population (ρW, ρFv, ρFm, ρM) in subtraction from respective populations' DEs.
- Insects as food source increases bird population, but at a smaller rate: ρ' multiplied by respective populations adds to *B* ($\rho' << \rho$)
    - Introduce *k* as net birth rate for *B*
        - Can use 62.5% average annual survival rate to inform *k*[12], with more research can be further developed to represent accurate birth rate
    - B has higher trophic level due to predator status, so assume smaller population: at t0, 10% of total insect population (750,000 butterflies + 200,000 wasps): *B(0)*= 95,000 birds. Impact ρ has on insect populations depends directly on size of *B*
    - B also has longer lifespan (3 years)[11], so model would have to consider differential lifespan (*k* should be small because birth rate *per day*)
    - B doesn't impact *E* directly but does have significant indirect impact on *E*

New Additions to Model



New Parameters:

k: combined birth and death rate of birds (net birth rate)

ϱ: number of insects birds eat per day

ϱ’: contribution of feeding on insects to bird population

$$\frac{dB}{dt} = kB + \frac{\rho' F_v}{F_m + F_v} + \frac{\rho' F_m}{F_m + F_v} + \rho' M + \rho' W$$

References

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No matter how hard I try
I can't eggscape the pain
HELLMANN'S

Questions?


fantasy
Is this THE REAL LIFE ?


help