Chemical Espionage

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Problem

- To model the population dynamics of a host-parasite type population of butterflies and wasps.
- Males release anti-aphrodisiacs to fend off other males so that the females can find a good place to lay their eggs which increases the probability of them hatching.
- The anti-aphrodisiac attracts parasites. The parasites latch onto the female and parasitize their eggs.

Lotka-Volterra Equations

Lotka-Volterra equations model systems of prey and predator populations. Below described is an example with rabbits as prey and foxes as predators.

$$\frac{dR}{dt} = \alpha R - \beta RF$$

$$\frac{dF}{dt} = \delta RF - \gamma F$$

Assumptions:

1. The prey population finds ample food at all times.
2. The food supply of the predator population depends entirely on the size of the prey population.
3. The rate of change of population is proportional to its size.
4. During the process, the environment does not change in favour of one species, and genetic adaptation is inconsequential.
5. Predators have limitless appetite.

Conceptualization of Method

- H ≡ population of butterflies (host), with Hm ≡ male population Hf ≡ female population,
- P ≡ population of wasps (parasite)

- **dH/dt = (rate at which hosts are born) - (rate of host's death)**
    - = (rate at which eggs are laid) * (fraction of eggs which hatch) - (rate of host's death)
- **dP/dt = (rate at which parasites grow) - (rate at which parasite's die)**

<u>Assumptions:</u>

- Hm > Hf : females are harassed and have lower probability of placing eggs in good spots. Therefore probability of hatching goes down
- An arbitrarily small Hm can fertilize an arbitrarily large Hf
    - Consequently, P depends only on Hf
- Both populations have natural death rate (age)

Mathematical Formulation

- rate of eggs being laid $= \alpha H_f$, $\alpha =$ growth rate of host
- fraction of eggs that hatch - depends on quality of nest and rate at which parasite eats butterfly eggs
- Goodness of nest - let $F_{nest} = 1$ if $\frac{F_f}{F_m} \geq 1$ and $F_{nest} = \frac{F_f}{F_m}$ otherwise
- Rate of consumption - let β be the rate at which the wasps consume butterfly eggs, then let $F_p = \frac{\beta}{P}$ if $\frac{\beta}{P} \leq 1$ and $F_p = 1$ otherwise
- Death rate - Define σ to be the natural death rate of the host
- With this, our model is

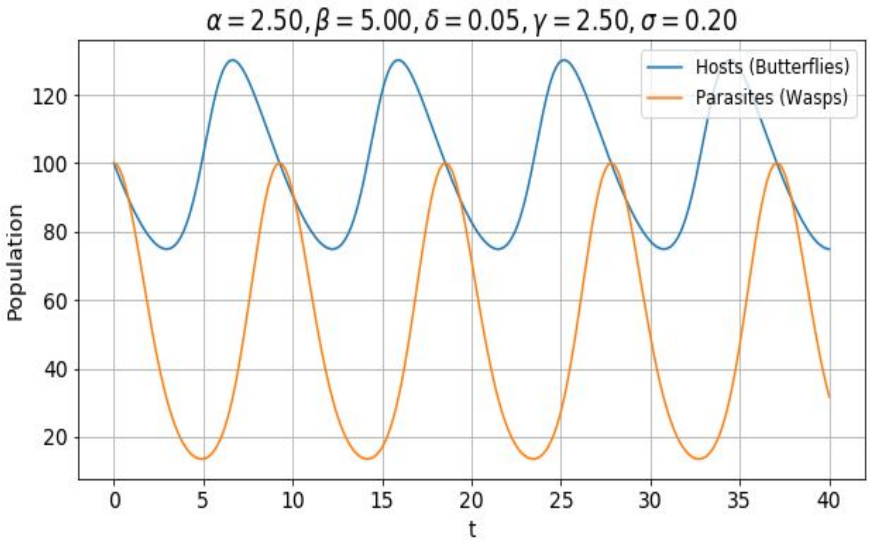
$$\frac{dH}{dt} = \alpha H_f (F_p)(F_n est) - \sigma H$$

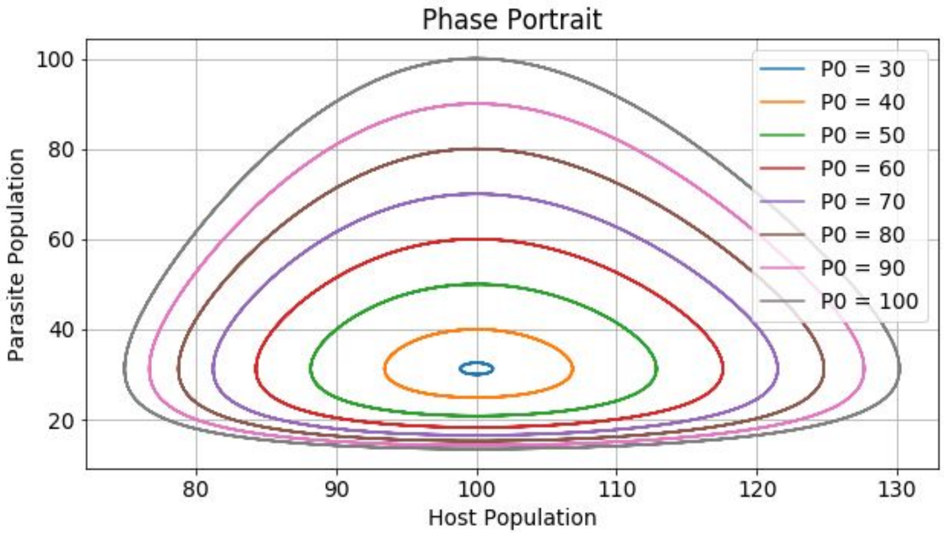
Mathematical Formulation

- Growth of parasite - let δ be the growth rate of the parasite, and γ be the death rate of the parasite
- $\frac{dP}{dt} = \delta H_f P - \gamma P$

Results

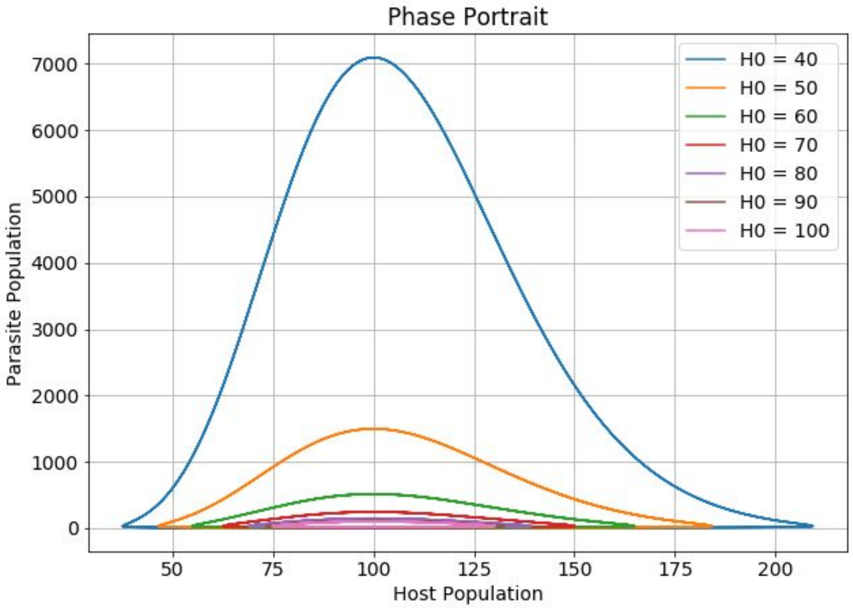
For random parameters and P0 = H0 = 100, with equal Hf/Hm ratio

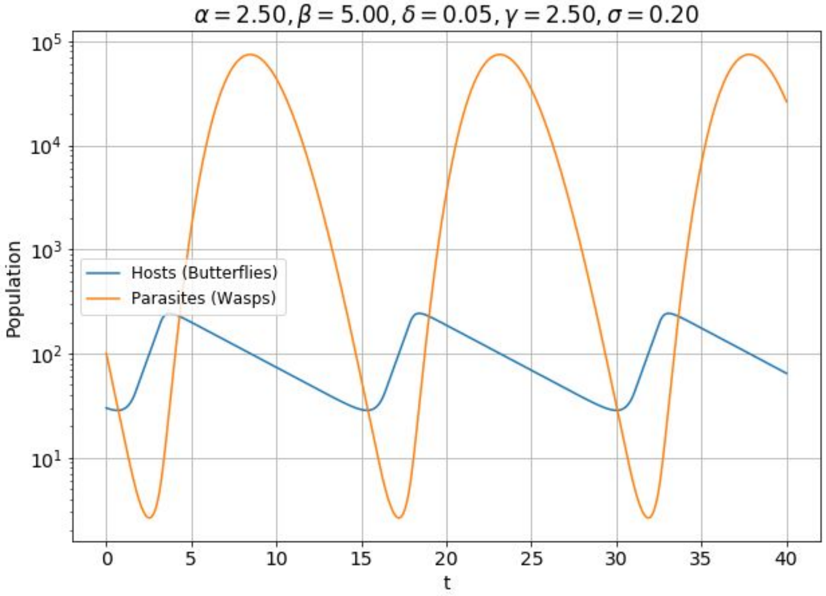




If we vary H0:

H0 = 30
P0 = 100



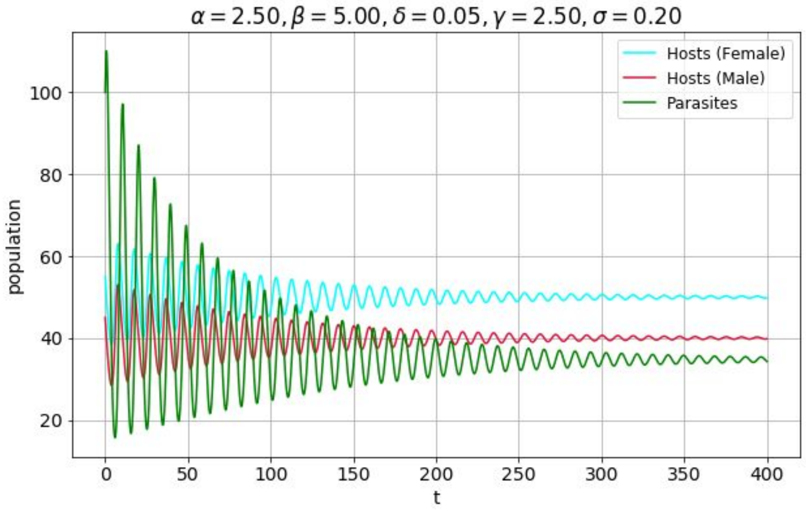


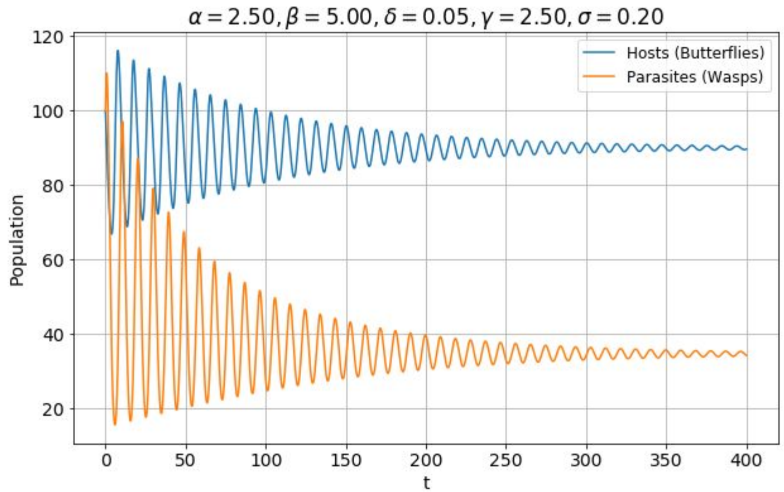
What if Hf/Hm != 1?

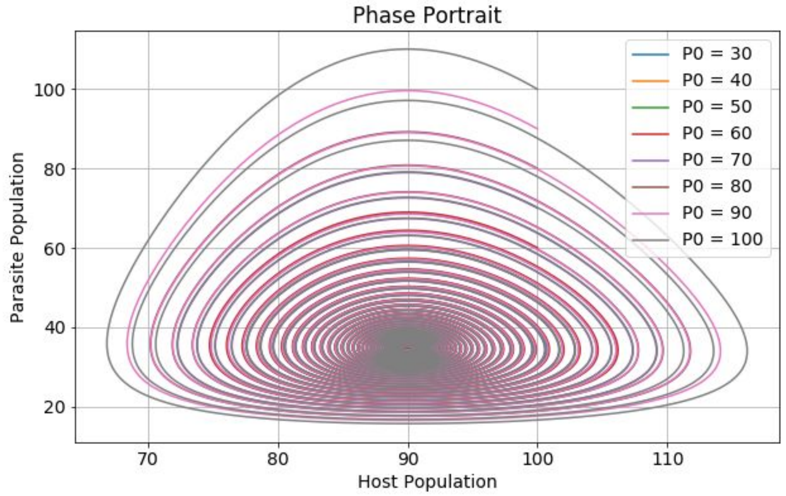
P0 = 100

Hf_0 = 55

Hm_0 = 45





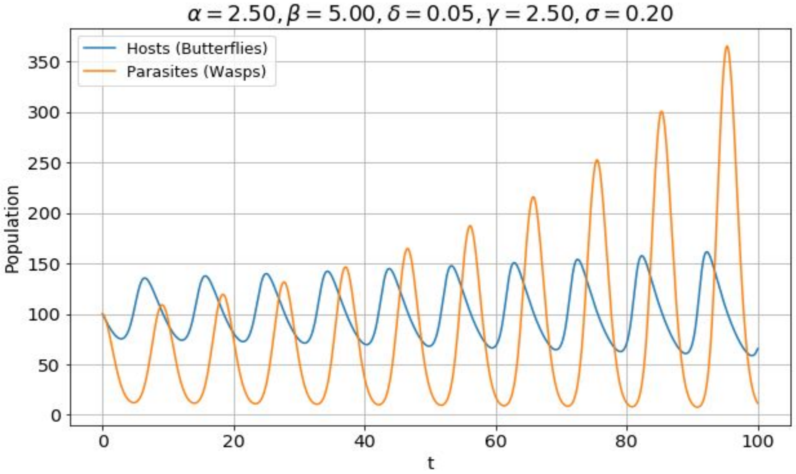


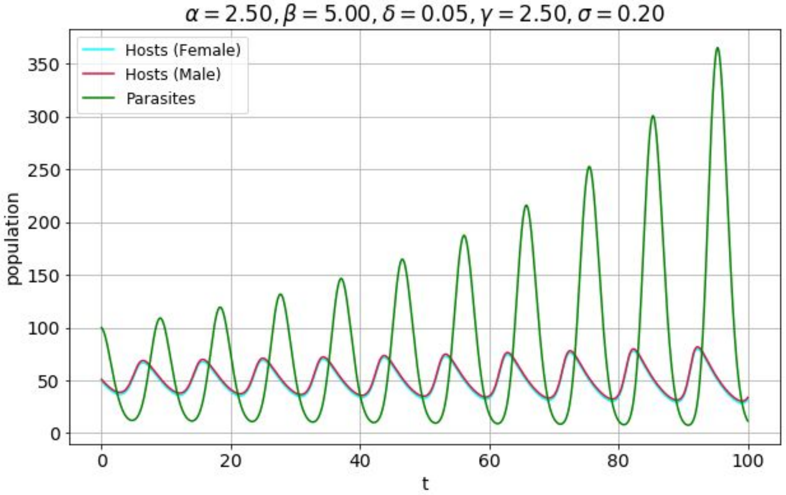
P0 = 100

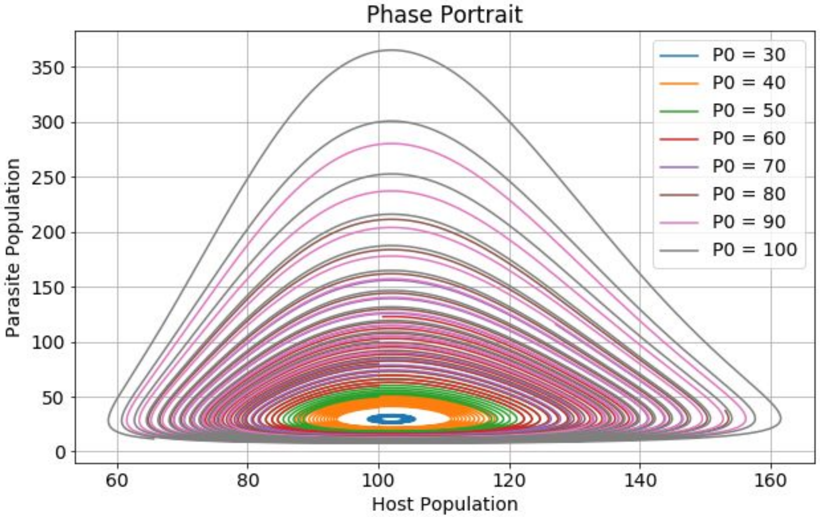
Hf_0 = 45

Hm_0 = 55

Reveals essential flaw in our model!

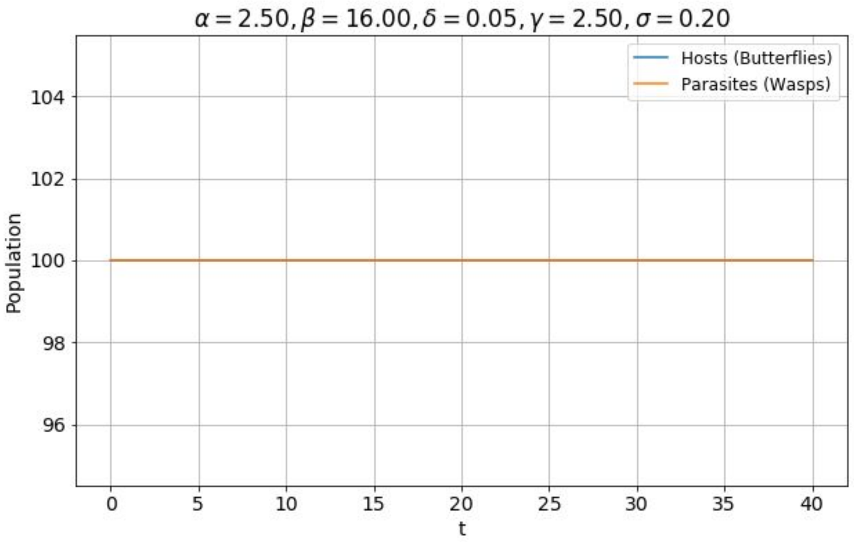


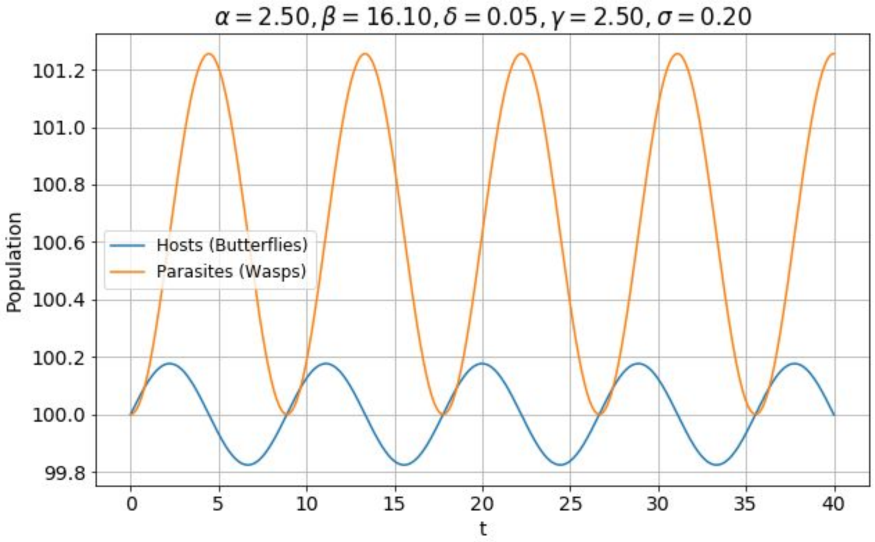




Does an equilibrium exist?

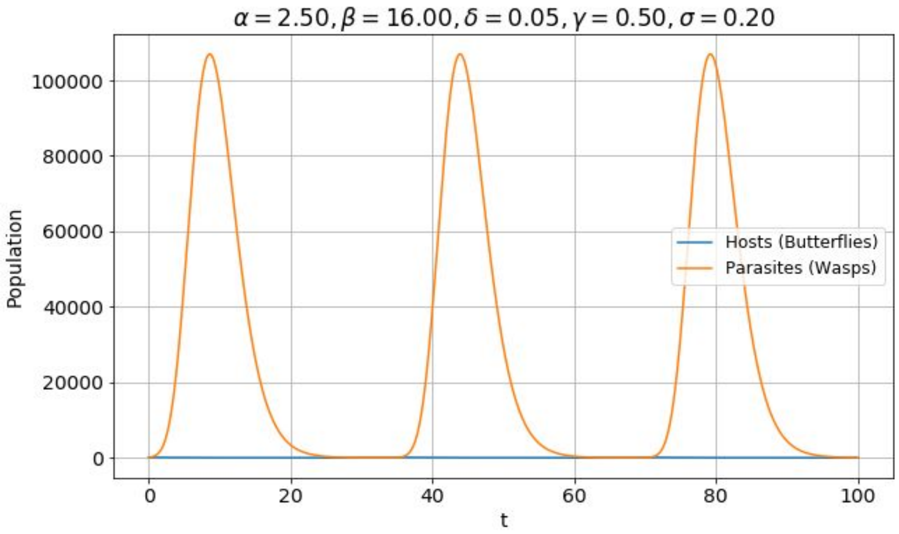
Can vary parameters to change population behavior. Through random variation, equilibrium parameters can be found.

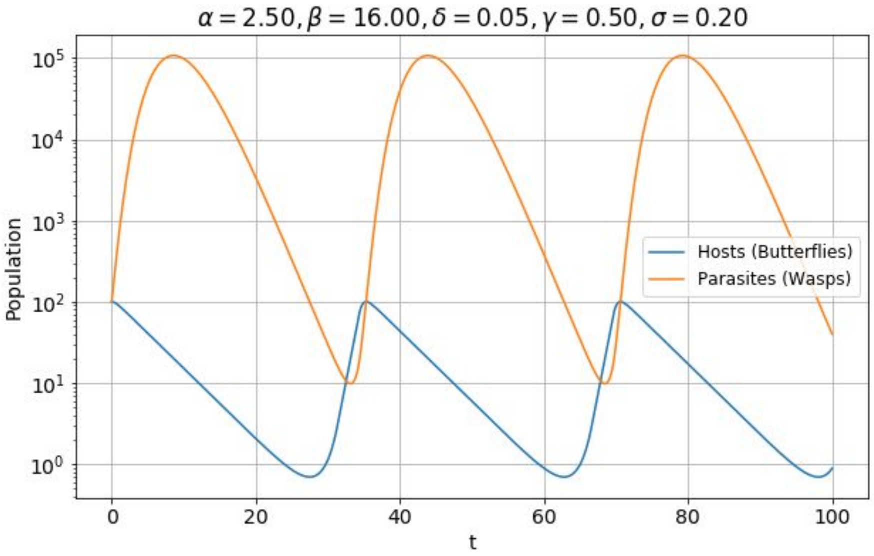




Extinction Risks

Change gamma (death rate of Parasites) from 2.5 to 0.5





Discussion

New Predator - Bird

Since, the birds eat both, the butterflies and the parasites, there would be a negative term added to both of the population equations which would be directly proportional to the population of the birds.

The rate of change of the bird population would depend upon the total food available i.e. the population of the (hosts + parasites) and also the natural death rate of the birds.

Equation for New Predatory Dynamics

- $\frac{dH}{dt} = \alpha H_f(F_p)(F_{nest}) - \sigma H - \omega_H B$
- $\frac{dP}{dt} = \delta H_f P - \gamma P - \omega_p B$
- $\frac{dB}{dt} = \mu(H + P) - \Omega B$
- New variables: ω_H, ω_p are the rate that the bird consumes the butterflies and the wasps, respectively, μ is the growth rate of the bird population
- Note - in this model, the $-\omega_H B$ term implies that the birds are eating the butterflies directly instead of eating their eggs (birds are predators not parasites)