PREDATOR-PREY MODEL OF WASPS, BUTTERFLIES, AND BIRDS

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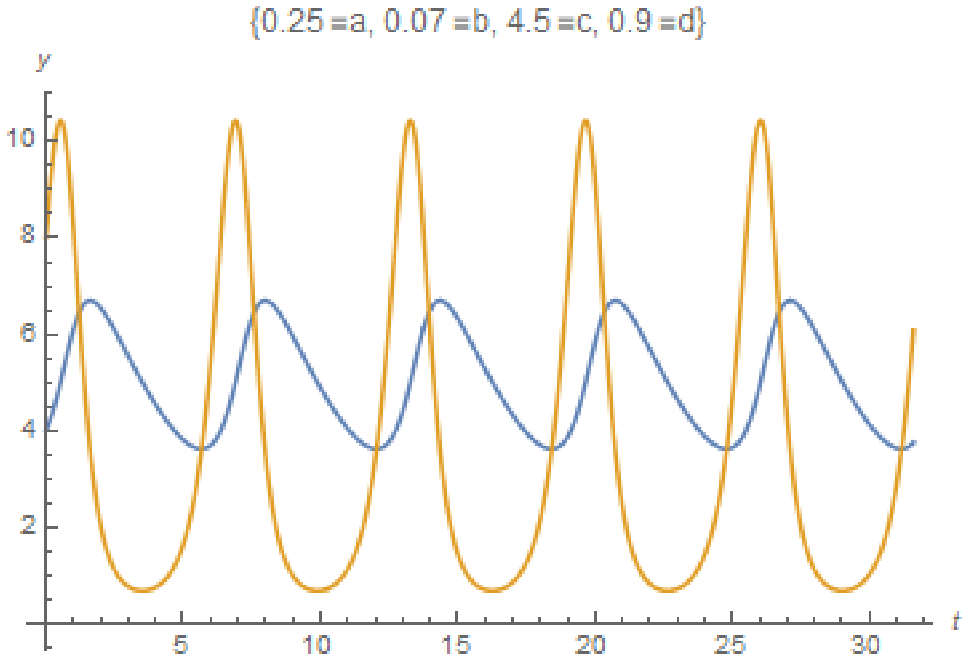
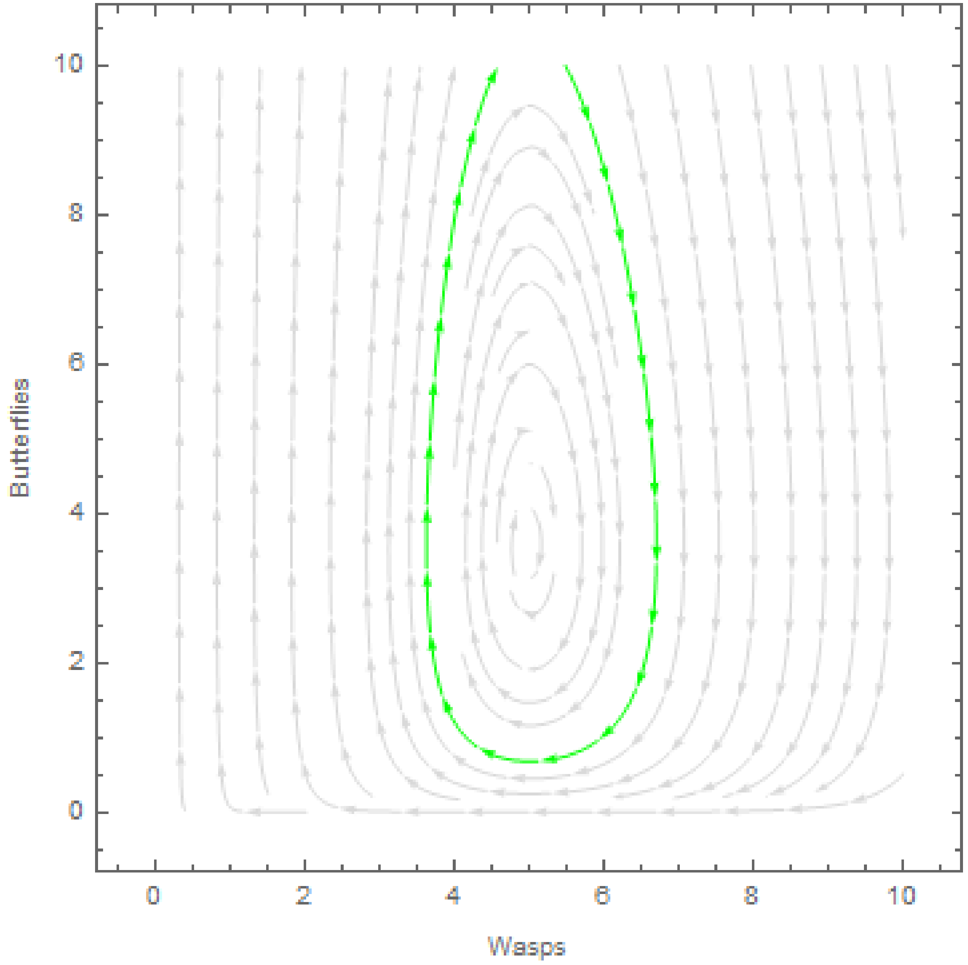
MODEL OF WASPS AND BUTTERFLIES

$$\frac{dx}{dt} = -ax + bxy \;, \frac{dy}{dt} = cy - dyx$$

```
 a=0.25;
b=0.07;
c=4.5;
d=0.9;
sol=NDSolve[{
    x'[t]==-a*x[t]+b*x[t]*y[t],
    y'[t]==c*y[t]-d*y[t]*x[t],x[0]==4,y[0]==8},{x[t],y[t]},{t,0,200}];
xa[t_]=x[t]/.First[sol];
ya[t_]=y[t]/.First[sol];
Manipulate[Plot[{xa[t],ya[t]},{t,0,tmax},
  PlotLabel->{"=a"a,"=b"b,"=c"c,"=d"d},
  AxesLabel->{t,y},
  PlotLegends->{"Wasps","Butterflies"}],
 {tmax,.01,200}]
```

RELATIONSHIP OF WASPS AND BUTTERFLIES



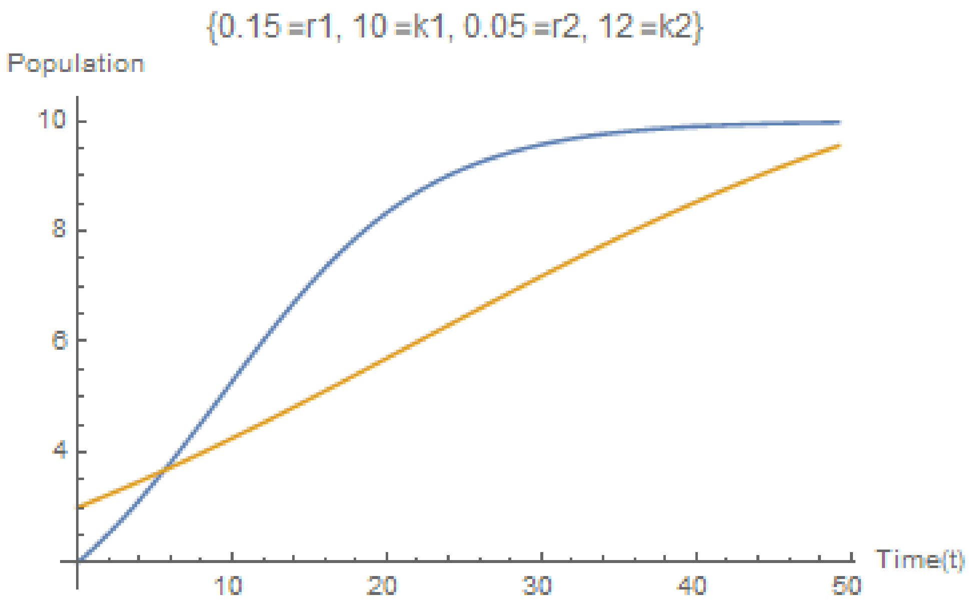


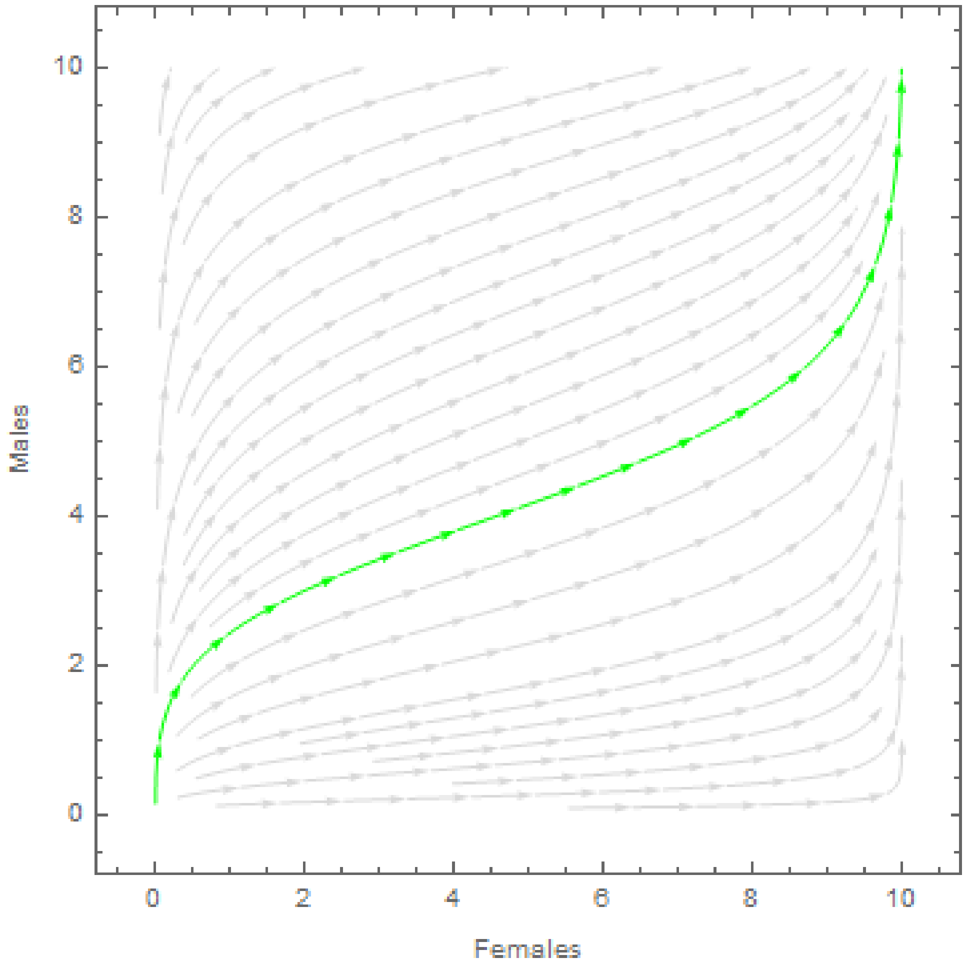
SOLVING LOGISTIC GROWTH FOR BUTTERFLIES

$$\frac{dP}{dt} = rP(1 - \frac{P}{K})$$

```
r1 = 0.15;
k1 = 10;
r2 = 0.05;
k2 = 12;
f0 = 2;
m0 = 3;
sol = NDSolve[{
    F'[t] == r1 * F[t] * (1 - F[t] / k1),
    m'[t] == r2 * m[t] * (1 - m[t] / k2), F[0] == f0, m[0] == m0}, {F[t], m[t]}, {t, 0, 100}];
Fa[t_] = F[t] /. First[sol];
ma[t_] = m[t] /. First[sol];
Manipulate[Plot[{Fa[t], ma[t]}, {t, 0, tmax},
  PlotLabel → {"=r1" r1, "=k1" k1, "=r2" r2, "=k2" k2},
  AxesLabel → {"Time(t)", "Population"},
  PlotLegends → {"Females", "Males"}],
 {tmax, .01, 100}]
StreamPlot[{r1 * x * (1 - x / k1), r2 * y * (1 - y / k2)}, {x, 0, 10}, {y, 0, 10},
 Axes → True,
 StreamStyle → LightGray,
 StreamPoints → {{{{f0, m0}, Green}, Automatic}},
 FrameLabel → {"Females", "Males"}]
```

RELATIONSHIP OF MALE AND FEMALE BUTTERFLIES





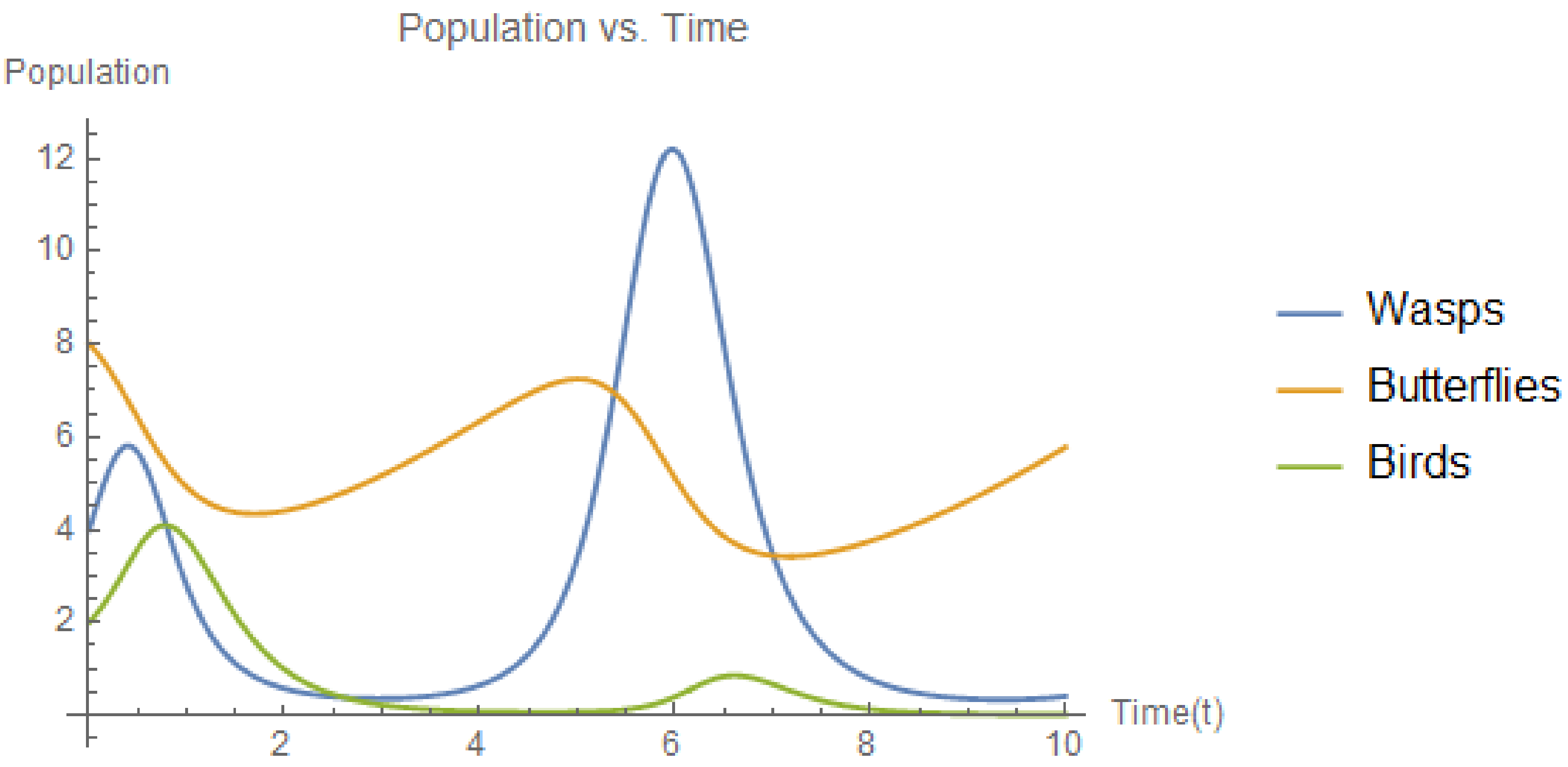
A NEW PREDATOR IS ADDED

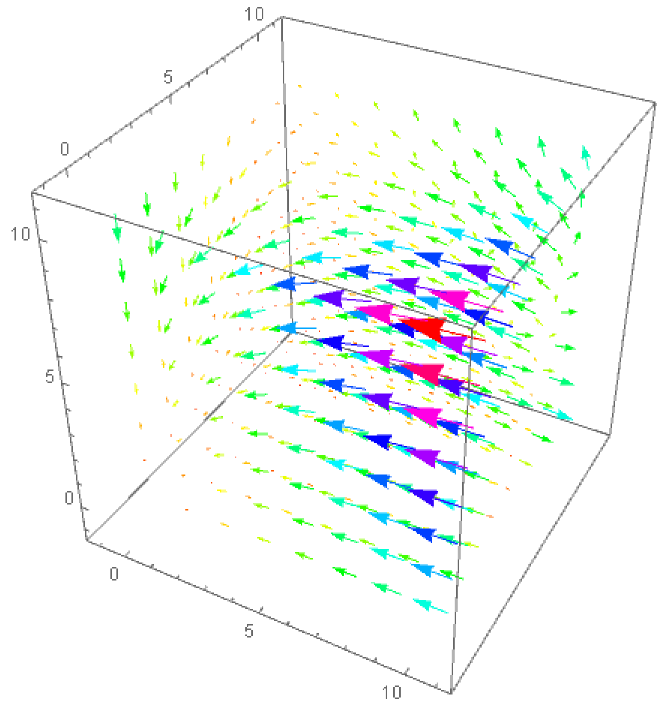
When a bird is added to the ecosystem, it attacks the populations of the wasps and the butterflies. The best way to approach this new problem is a sum predator-prey models, accounting for all factors on the growth or decay of a species.

$$\frac{dx}{dt} = ax + byx - czx$$
$$\frac{dy}{dt} = dy - exy - fzy$$
$$\frac{dz}{dt} = -gz + hxz + iyz$$

MODEL OF THE FOOD CHAIN IN AN ECOSYSTEM

```
a = 0.25;
b = 0.07;
c = 0.12;
d = 4.5;
e = 0.9;
f = 0.49;
g = 3.8;
h = 0.46;
i = 0.31;
x0 = 4;
y0 = 8;
z0 = 2;
sol = NDSolve[{
    x'[t] == -d * x[t] + e * x[t] * y[t] - f * x[t] * z[t],
    y'[t] == a * y[t] - b * y[t] * x[t] - c * y[t] * z[t],
    z'[t] == -g * z[t] + h * z[t] * y[t] + i * z[t] * x[t],
    x[0] == x0, y[0] == y0, z[0] == z0}, {x[t], y[t], z[t]}, {t, 0, 200}];
xa[t_] = x[t] /. First[sol];
ya[t_] = y[t] /. First[sol];
za[t_] = z[t] /. First[sol];
```





RELATIONSHIP OF WASPS, BUTTERFLIES, AND BIRDS

ANALYSIS

There is small equilibrium value that allows the three variable predator-prey relationship to cycle. In the model the bird population goes off to zero, even when adjusting the rate relationships with the butterflies and wasps. The relationship between the birds and the wasps remain cyclic.