

A stylized, light green illustration of a plant with several leaves and a cluster of small, round buds or flowers, positioned on the left side of the slide.

CHEMICAL ESPIONAGE (PROBLEM C)

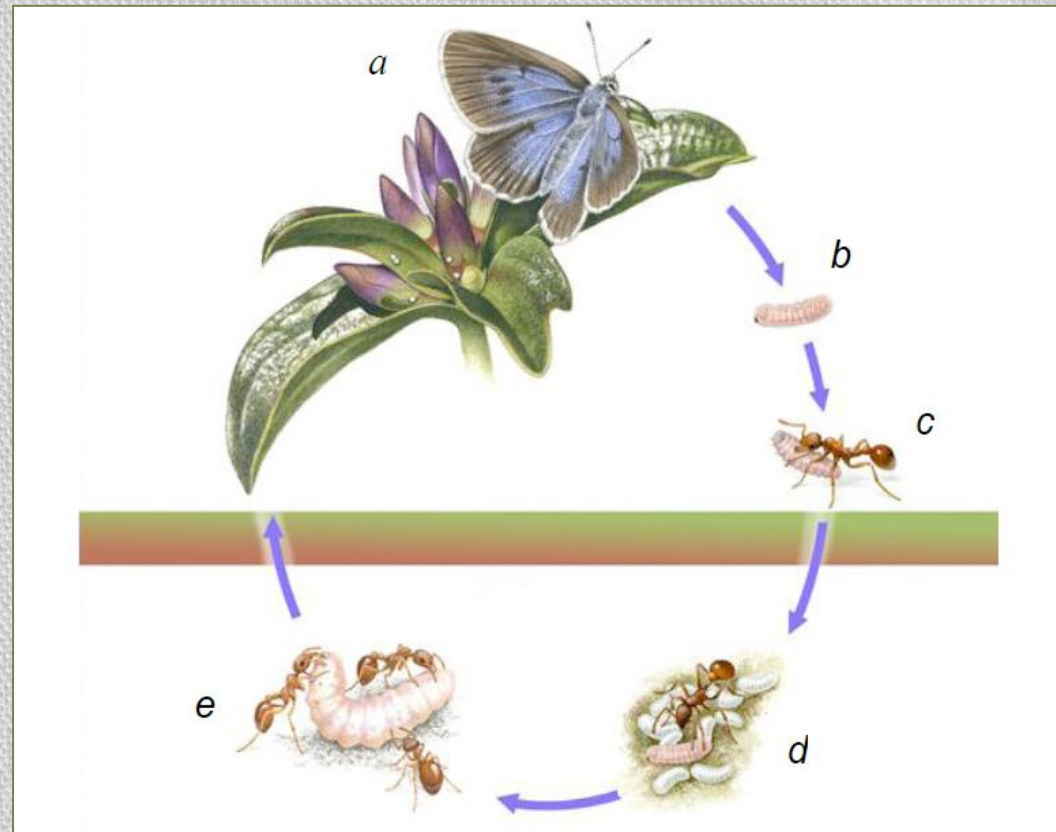
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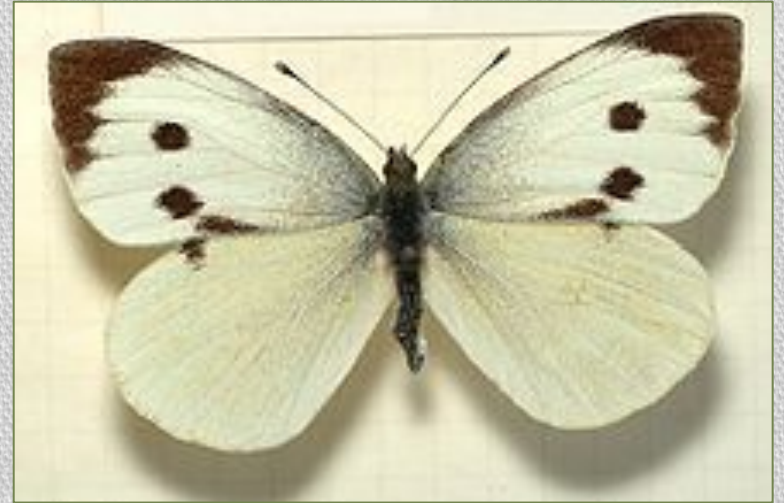
Background

- *Pieris brassicae*
- Chemical Signals
 - Aphrodisiacs
 - Anti-aphrodisiacs
- Parasitic Wasps



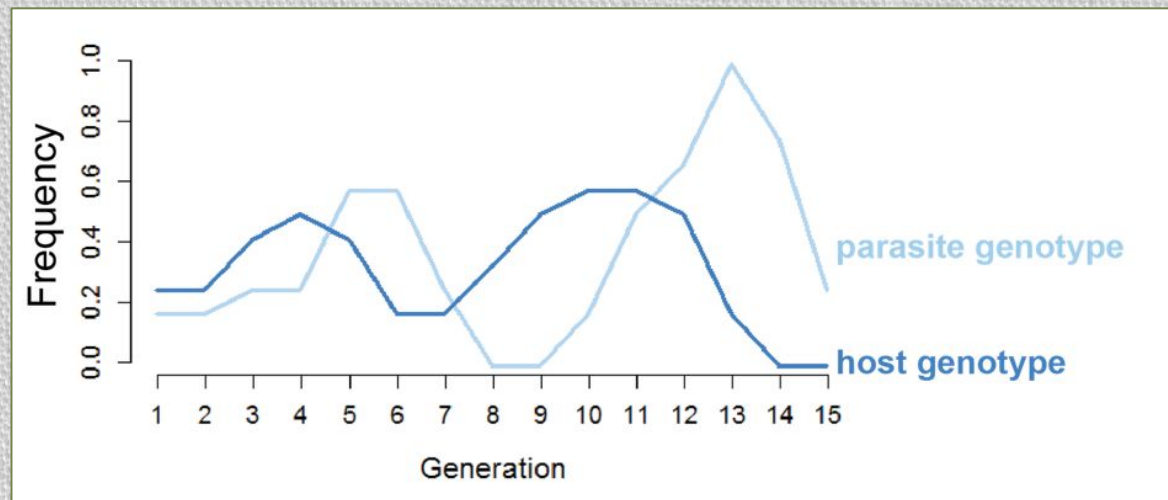
Assumptions

- Y-linked Inheritance
- Presence of Null Male *P. brassicae*
- 1:1 Sex Ratio
- No Variable Population Depressants
 - Weather
 - Predation
 - Starvation
- Total Mortality of Parasitized Eggs



Rationale

- Lotka-Volterra Predator-Prey Equations
- Red-Queen Hypothesis



Variables Definition	Variable	Value
Population of butterflies	P_N	$P_0 = 1000$
Population of non anti-aphrodisiac (NA) male butterflies	M_{NA}	$M_{NA} = 475$
Population of non anti-aphrodisiac NA male butterflies	M_{AA}	$M_{AA} = 25$
Population of wasps	w_N	$w_0 = 50$
Brood size	b	141
Survival to adulthood of butterfly population	P_s	0.42
Survival ratio of offspring of NA male butterflies	S_{NA}	0.65
Survival ratio of offspring of anti-aphrodisiac AA male butterflies	S_{AA}	0.95
Chance of parasitic wasp finding eggs if female mates with NA male	i_{NA}	0.01
Chance of parasitic wasp finding eggs if female mates with AA male	i_{AA}	0.025
Death rate of butterfly population	d_p	0.5
Death rate of wasp population	d_w	0.7

The System of Equations

$$(dP_N)/(dt) = [(dP_{AA})/(dt) + (dP_{NA})/(dt)]$$

$$(dP_{NA})/(dt) = S_p (S_{NA} * M_{NA(N-1)} * b [1 - w * i_{NA}]) - dP_{NA(N-1)}$$

$$(dP_{AA})/(dt) = S_p (S_{AA} * M_{AA(N-1)} * b [1 - w * i_{AA}]) - dP_{AA(N-1)}$$

$$(dM_{NA})/(dt) = (dP_{NA}/dt) * (1/2)$$

$$(dM_{AA})/(dt) = (dP_{AA}/dt) * (1/2)$$

$$(dw)/(dt) = w * b * [(i_{NA} * M_{NA(N-1)}) + (i_{AA} * M_{AA(N-1)})] - d_w * w_{(N-1)}$$

Reflections & Improvements

- Population Record
- Environmental Factors: flowers, lands, weather
- Oviposition Plant Preference
- Genetic Inheritance



Additional Issues

Problem C1

(Suppose you are asked to add an animal that is a predator of both the butterflies and the wasps, a bird for example. How would you change your model to accommodate this new situation?)

Additional Assumptions

- 1) Bird finds most advantageous plant (cabbage)
- 2) Least advantageous plant is sarson
- 3) To maximize nutritional gain versus energy loss, birds target cabbage plants (double oviposition)
- 4) The likelihood that the bird encounters/searches for larvae is affected by whether the larvae is on the cabbage
- 5) Females that mate with AA males have more time to search for cabbage
- 6) AA-sired larvae are therefore disproportionately represented on cabbage

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New Terms

Variables	Variables Defined
B_{arb}	Bird population
d_{arb}	Death by bird
a	likelihood bird encounters larvae
h	nutrition factor
v	Bird breeding factor
e	eggs

$$\frac{dP_{AA}}{dt} = S_p (S_{AA} * M_{AA(N-1)} * e * [1 - w_{(N-1)} * i_{AA}]) * (1 - d_{Brb} * B_{N-1} * a_{AA}) - dP_{AA(N-1)}$$

$$\frac{dP_{NA}}{dt} = S_p (S_{NA} * M_{NA(N-1)} * e * [1 - w_{(N-1)} * i_{NA}]) * (1 - d_{Brb} * B_{N-1} * a_{NA}) - dP_{NA(N-1)}$$

$$\frac{dw}{dt} = w_{(N-1)} * e * [(1 - d_{Brb,w} * B_{N-1} * a_{NA})] + [(M_{AA} * i_{AA}) (1 - d_{Brb,w} * B_{N-1} * a_{AA})] - d_w * w_{(N-1)}$$

$$\frac{dB_{Brb}}{dt} = v * B_{N-1} * [(h_p * d_b * e * (a_{AA} * M_{AA(N-1)} + a_{NA} * M_{NA(N-1)})) + (h_w * w_{n-1} * (a_{AA} * i_{AA} + a_{NA} * i_{NA}))] - (d_{Brb} * B_{N-1})$$

References

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- Morran LT, Schmidt OG, Gelarden IA, Parrish RC, Lively CM. 2011. Running with the Red Queen: Host-parasite coevolution selects for biparental sex. Science. 333(6039): 216-218.
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