Chemical Espionage

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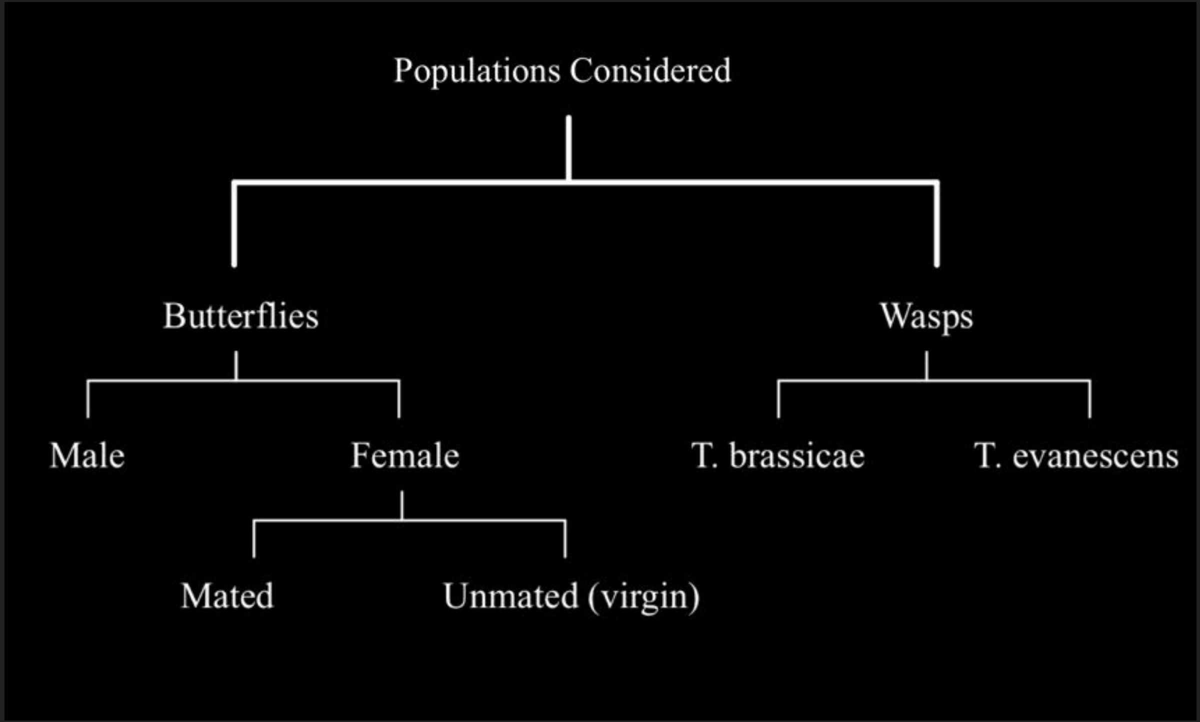
The Model: Description

We consider five populations:

- **P. brassicae**:
  - Male
    - M
  - Female
    - F_M
    - F_U
- **Wasps**:
  - T. brassicae
    - T
  - T. evanescens
    - E



Problem Statement & Assumptions:

Problem Statement:

We modeled the population trends of the butterfly *Pieris brassicae*, who host the parasitic wasps *Trichogramma brassicae* and *Trichogramma evanescens*.

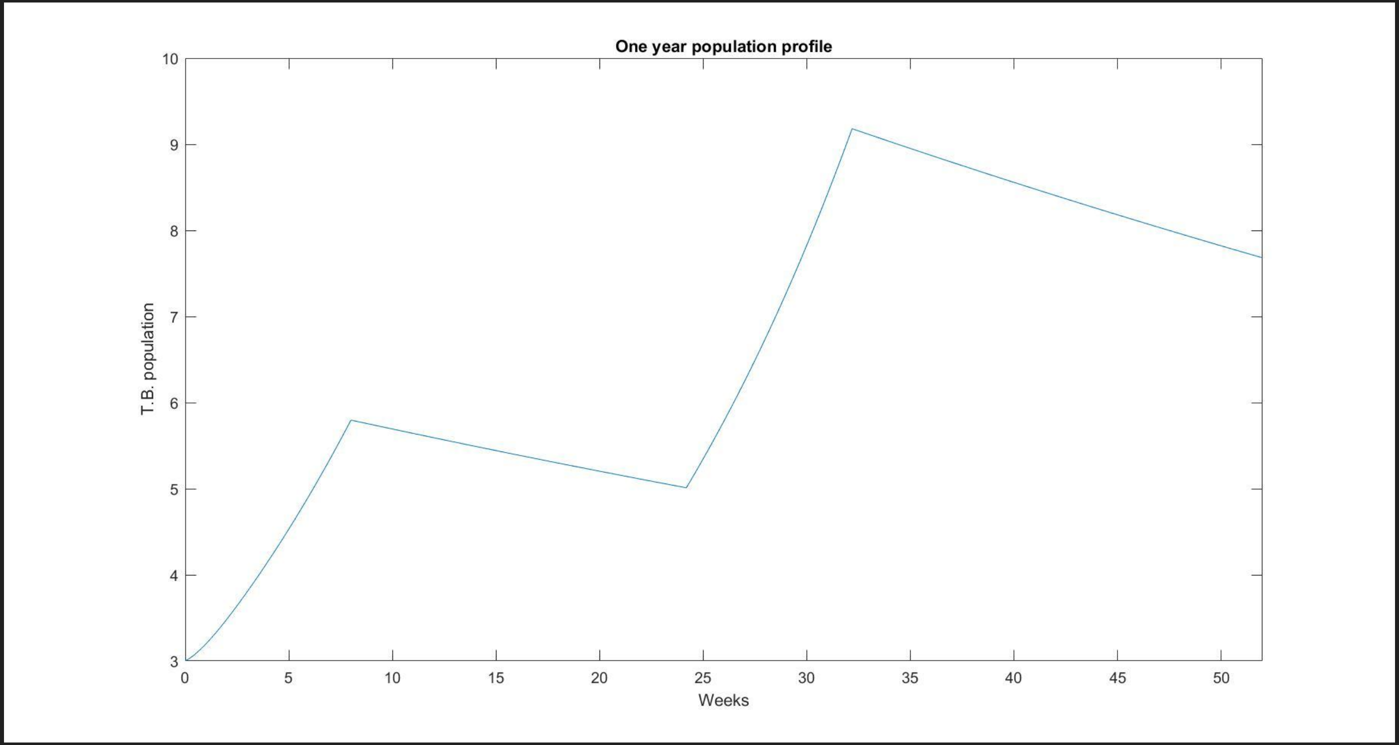
The male *P. brassicae*, after mating, emit an anti-aphrodisiac to protect their mate from the harassment of other males. This anti-aphrodisiac has the unintended effect of attracting the parasitic wasps, who use the chemical signal to identify and mount female *P. brassicae*. The wasps lay their eggs inside the *P. brassicae* eggs, and upon hatching, the wasps consume the butterfly eggs, killing the butterflies.

Assumptions:

1: *T. brassicae* and *T. evanescens* can parasitize the same clutches.

2. All three species have the same rate of survival.

Life Cycle


One year population profile
T.B. population
Weeks
10
9
8
7
6
5
4
3
0
5
10
15
20
25
30
35
40
45
50

Variables Defined:

Term	Definition	Symbol
Male *P. brassicae*	# of male *P. brassicae* individuals	M
Mated female *P. brassicae*	# of mated female *P. brassicae* individuals	F_M
Unmated female *P. brassicae*	# of unmated female *P. brassicae* individuals	F_U
T. brassicae	# of *T. brassicae* individuals	TB
T. evanescens	# of *T. evanescens* individuals	TE

The Model: Differential Equations

$$\frac{dM}{dt} = (Male\ P.B.\ hatchlings) - (Death\ constant) * M$$

$$\frac{dF_u}{dt} = (Female\ P.B.\ hatchlings) - (Death\ constant) * F_u - (Rate\ at\ which\ females\ mate)$$

$$\frac{dF_m}{dt} = (Rate\ at\ which\ females\ mate) - (Death\ constant) * F_m$$

$$\frac{dTB}{dt} = (T.B.\ hatchlings) - (Death\ constant) * TB$$

$$\frac{dTE}{dt} = (T.E.\ hatchlings) - (Death\ constant) * TE$$

The Model: Differential Equations (Physical constants)

Term	Definition	Derivation
SR	Proportion of eggs that successfully mature	0.0074 organisms per week
DR	Death rate	0.03 organisms per week (mating season); 0.01 organisms per week (off season);
p_{TB}	Probability parasite wasp *T.B.* does not mount a *P.B.* butterfly	0.9844
p_{TE}	Probability parasite wasp *T.E.* does not mount a *P.B.* butterfly	0.8524
E_X	Number of eggs per brood per week for species *X*	30.63 for *P.B.* 5.73 for *T.B.* 4.78 for *T.E.*
A_{0TB}	Probability a *P.B.* butterfly is not mounted by a specific *T.B.* wasp (no aphrodisiac)	0.9869
A_{1TB}	Probability a *P.B.* butterfly is not mounted by a specific *T.B.* wasp (aphrodisiac)	0.9844
A_{0TE}	Probability a *P.B.* butterfly is not mounted by a specific *T.E.* wasp (no aphrodisiac)	0.8954
A_{1TE}	Probability a *P.B.* butterfly is not mounted by a specific *T.E.* wasp (aphrodisiac)	0.8524
P_M	Proportion of *T.B.* that are male	0.5745
P_F	Proportion of *T.B.* that are female	0.4255

The Model: Differential Equations

$$\frac{dM}{dt} = ({P_{TB}}^{TB})({P_{TE}}^{TE})(F_M)(eggs)(SR)(p_m) - (DR)(M)$$

$$\frac{dF_U}{dt} = ({P_{TB}}^{TB})({P_{TE}}^{TE})(F_M)(eggs)(SR)(p_f) - \frac{(F_U * M)}{(F_M)} - (DR)(F_M)$$

$$\frac{dF_M}{dt} = \frac{(F_U * M)}{F_M} - (DR)(F_M)$$

$$\frac{dTB}{dt} = (1 - {P_{TB}}^{TB})(F_M)(SR)(TB)(Eggs_{TB}) - (DR)(TB)$$

$$\frac{dTE}{dt} = (1 - {P_{TE}}^{TE})(F_M)(SR)(TE)(Eggs_{TE}) - (DR)(TE)$$

The Model: Equation Justifications

P. brassicae eggs surviving to adulthood

(P_{TE}) :	The probability that one P. brassicae isn't mounted by a TE individual
$(P_{TE}{}^{TE})$:	The probability that one P. brassicae isn't mounted by *any* TE individual
$(P_{TB}{}^{TB})(P_{TE}{}^{TE})$:	The probability that one P. brassicae isn't mounted by *any* wasp
$(P_{TB}{}^{TB})(P_{TE}{}^{TE}) * F_M$:	The number of unmounted mated females
$(P_{TB}{}^{TB})(P_{TE}{}^{TE})(F_M)(eggs)(SR)$:	The number of P. brassicae eggs surviving to adulthood

The Model: Equation Justifications (cont.)

Wasp eggs surviving to adulthood (using T. evanescens as example)

$({P_{TE}}^{TE})$:	The probability that one P. brassicae isn't mounted by any TE individual
$(1 - {P_{TE}}^{TE})$:	The probability that one P. brassicae *is* mounted by any TE individual
$(1 - {P_{TE}}^{TE})(F_M)(TE)$:	The # of interactions between the P. brassicae and T. evanescens populations
$(1 - {P_{TE}}^{TE})(F_M)(TE)(SR)(Eggs)$:	The number of viable T. evanescens eggs

The Model: Equation Justifications (cont.)

Mating rate of butterflies:

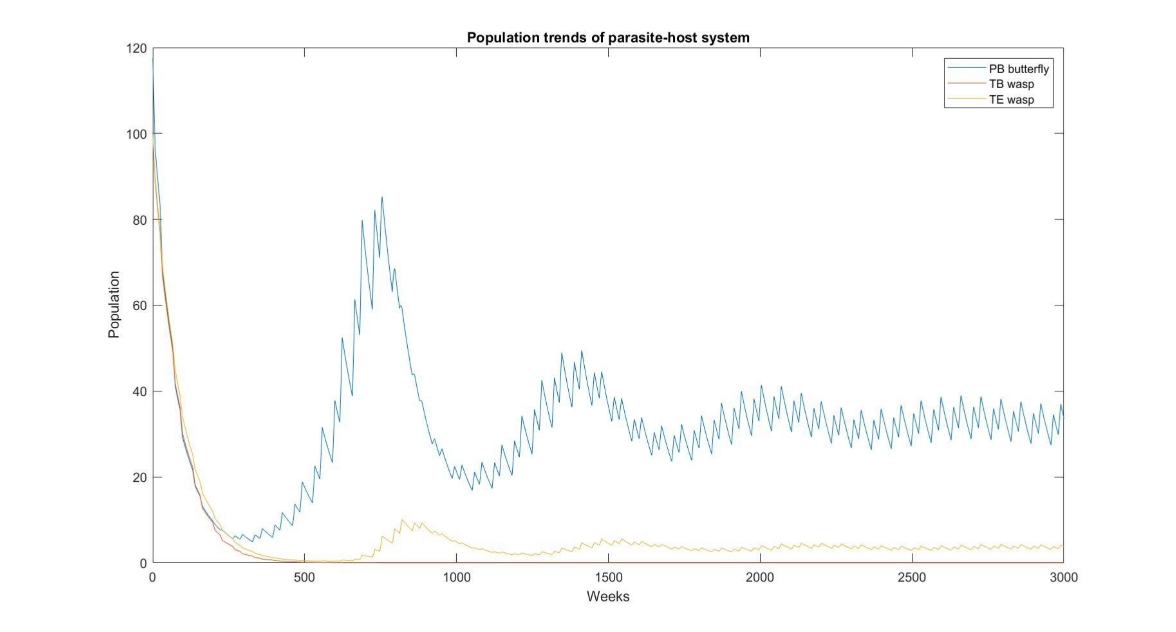
$(F_U * M)$:

- Rate at which unmated females encounter the male butterflies

$\frac{(F_U * M)}{(F_M)}$:

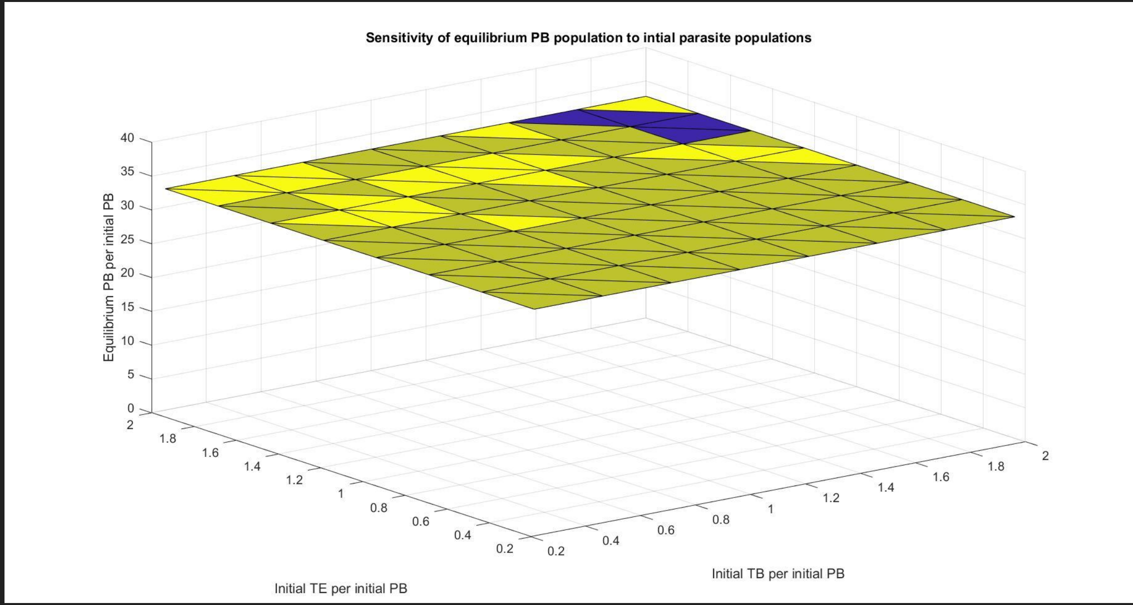
- F_M is the denominator because the more mated females, the more anti-aphrodisiac, which discourages further mating

Results



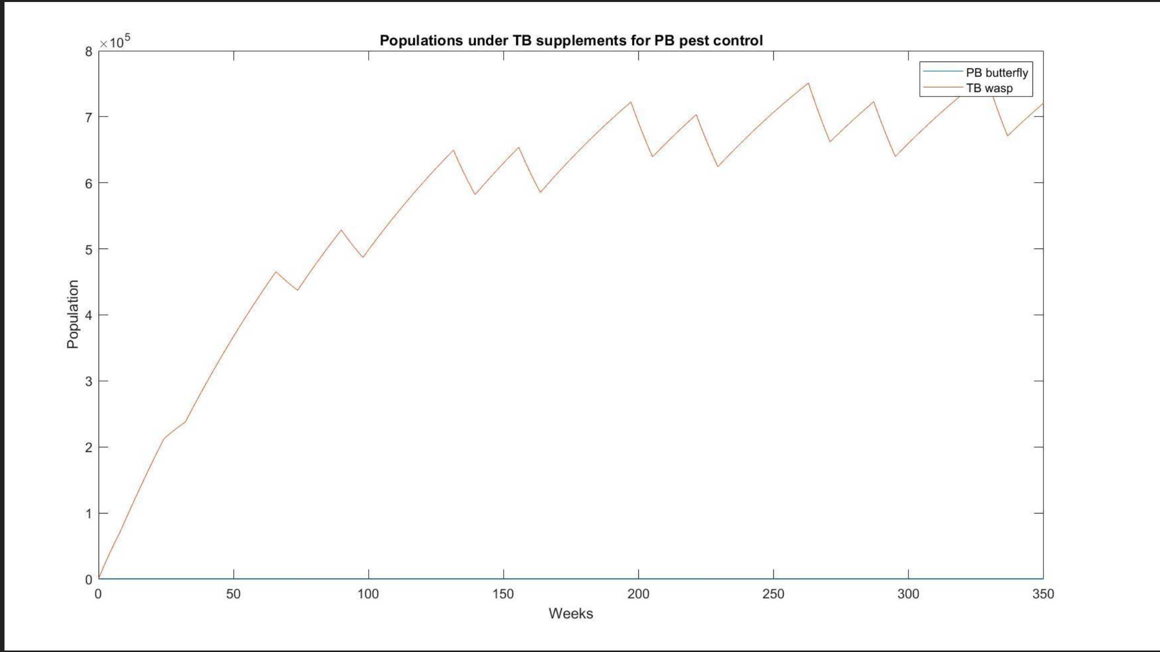
Initial populations of male *P. brassicae*, unmated female *P. brassicae*, mated female *P. brassicae*, *T. brassicae*, and *T. evanescens* are equal to 63, 45, 5, 100, and 100, respectively.

Sensitivity Test Results:



The total population of *P. brassicae, T. brassicae,* and *T. evanescens* will always converge to 32.74%, 0.00%, and 3.54% of the initial *P .brassicae* population, respectively, as long as all initial populations are positive (which is always expected).

Farmer Application



The farmer supplements 10,000 *T. brassicae* eggs per week to control the population of *P. brassicae* pests, as directed by commercial anti-pest procedures

The New Model: Problem Statement

Find how the model changes if unmated female *T. brassicae* can detect a male's propensity for emitting anti-aphrodisiacs.

Assume that female individuals will discriminate against potential male mates depending on their propensity for emitting anti-aphrodisiacs.

The Model: Differential Equations

$$\frac{dM}{dt} = (P_{TB}{}^{TB})(P_{TE}{}^{TE})(F_M)(eggs)(SR)(p_m) - (DR)(M)$$

$$\frac{dF_U}{dt} = (P_{TB}{}^{TB})(P_{TE}{}^{TE})(F_M)(eggs)(SR)(p_f) - \frac{(F_U * M)}{(F_M)} - (DR)(F_M)$$

$$\frac{dF_M}{dt} = \frac{(F_U * M)}{F_M} - (DR)(F_M)$$

$$\frac{dTB}{dt} = (1 - P_{TB}{}^{TB})(F_M)(SR)(TB)(Eggs_{TB}) - (DR)(TB)$$

$$\frac{dTE}{dt} = (1 - P_{TE}{}^{TE})(F_M)(SR)(TE)(Eggs_{TE}) - (DR)(TE)$$

Adjustments to Current Model

Term	Definition	Value
c	A *P.B.* butterfly's propensity for emitting anti-aphrodisiacs	c=0 means no emission, c=1 means normal emission c>1 means high emission
A_{0TB}	Probability a *P.B.* butterfly is not mounted by a specific *T.B.* wasp (no aphrodisiac)	0.9869
A_{1TB}	Probability a *P.B.* butterfly is not mounted by a specific *T.B.* wasp (aphrodisiac)	0.9844

$$\textit{Rate at which females mate}: \frac{F_u M}{F_m} \rightarrow \frac{F_u M}{(F_m)^c}$$

$$\textit{Probability a P.B. butterfly is not mounted by a T.B. wasp}: A_{1TB} \rightarrow A_{0TB} + c * (A_{1TB} - A_{0TB}); \quad A_{1TB} > A_{0TB}$$

Similar equation change for *T.E.* wasp mounting probability.

The New Model: Differential Equations

$$\frac{dM}{dt} = ({P_{TB}}^{TB})({P_{TE}}^{TE})(F_M)(eggs)(SR)(p_m) - (DR)(M)$$

$$\frac{dF_U}{dt} = ({P_{TB}}^{TB})({P_{TE}}^{TE})(F_M)(eggs)(SR)(p_f) - \frac{(F_U * M)}{{F_m}^c} - (DR)(F_M)$$

$$\frac{dF_M}{dt} = \frac{(F_U * M)}{{F_M}^c} - (DR)(F_M)$$

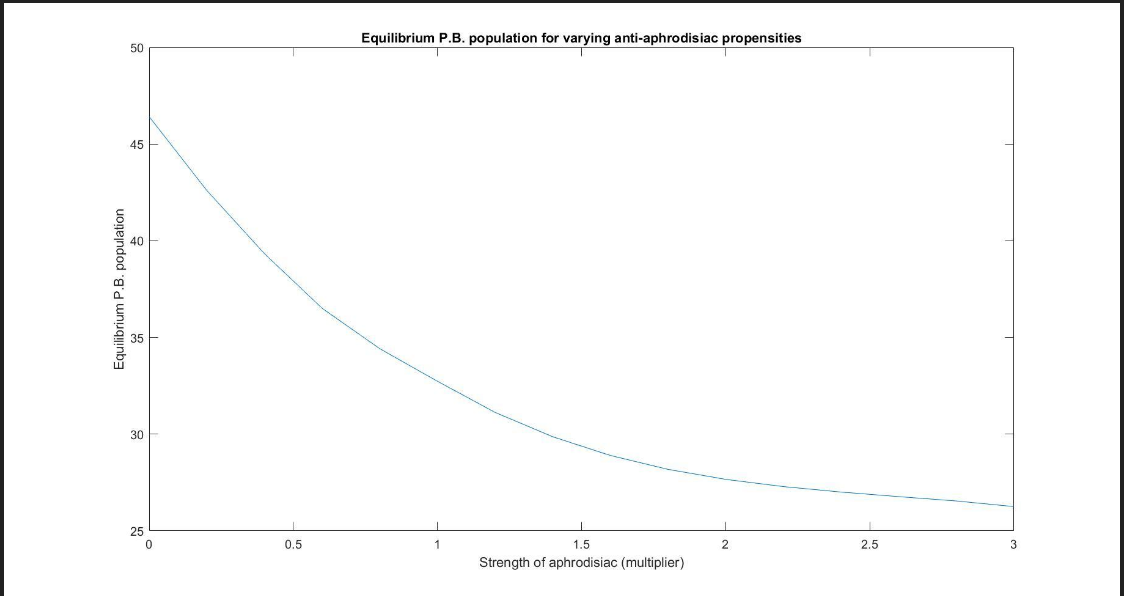
$$\frac{dTB}{dt} = (1 - {P_{TB}}^{TB})(F_M)(SR)(TB) - (DR)(TB)$$

$$\frac{dTE}{dt} = (1 - {P_{TE}}^{TE})(F_M)(SR)(TE) - (DR)(TE)$$

$$P_{TB} = A_{0TB} + (A_{1TB} - A_{0TB})c$$

$$P_{TE} = A_{0TE} + (A_{1TE} - A_{0TE})c$$

The New Model: Results



Assumption: female butterflies will tend to select males whose anti-aphrodisiac propensities favor long term population growth rates

Conclusion

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Absence of certain Wasps