Modeling Chemical Espionage

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Chemical Espionage

Problem:
What are the trade-offs and balance between the Large white cabbage butterfly (Pieris brassicae), its use of anti-aphrodisiacs, and the interaction between two species of parasitic wasp, Trichogramma brassicae and Trichogramma evanescens.

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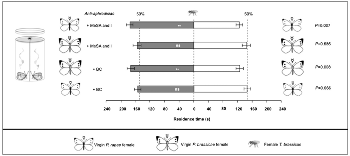
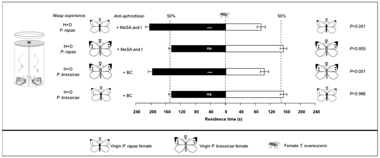
- $P_1 =$ Population of *P. Brassicae* eggs fertilized without BC
- $\beta =$ Birth Rate of population
- $\lambda =$ Death Rate of population



(a) *T. brassicae*



(b) *T. evanescens*

Figure: Residence times, for σ and ρ

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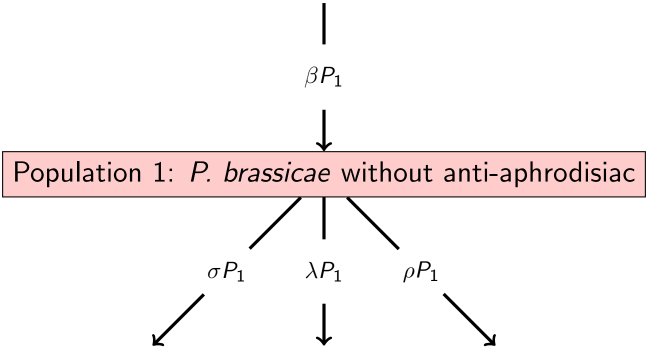

- $\sigma =$ rate at which *T. evanescens* consumes eggs of P_1
- $\rho =$ rate at which *T. brassicae* consumes eggs of P_1

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Considering the previous system constructed, we produce the following equation based upon the population of *P. brassicae* reproducing without the anti-aphrodisiac.

$$\begin{aligned} \Delta P_1 &= \beta P_1 - \lambda P_1 - \sigma P_1 - \rho P_1 \\ &= \beta P_1 - (\lambda + \sigma + \rho) P_1 \end{aligned}$$

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In what way does the presence of the anti-aphrodisiac benzyl cyanide (BC) affect the rate at which the two species of parisitic wasps consume *P. brassicae*?

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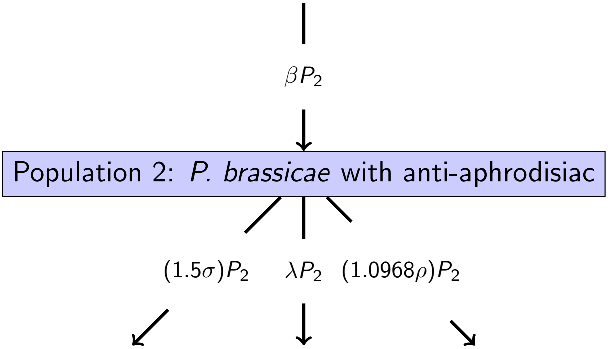
- The residence time of *T. brassicae* with the anti-aphrodisiac is approximately 1.0968 times the residence time without the anti-aphrodisiac BC [1, 2].
- The residence time of *T. evanescens* with the anti-aphrodisiac is approximately 1.5 times the residence time without the anti-aphrodisiac BC [1, 2].



- $1.5\sigma =$ rate at which *T. evanescens* consumes eggs of P_2
- $1.0968\rho =$ rate at which *T. brassicae* consumes eggs of P_2

Chemical Espionage

Considering the previous system constructed, we produce the following equation based upon the population of *P. brassicae* reproducing with the anti-aphrodisiac.

$$\begin{aligned}\Delta P_2 &= \beta P_2 - \lambda P_2 - 1.5\sigma P_2 - 1.0968\rho P_2 \\ &= \beta P_2 - (\lambda + 1.5\sigma + 1.0968\rho) P_2\end{aligned}$$

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- P_1 = Population of *P. Brassicae* eggs fertilized without BC
- P_2 = Population of *P. Brassicae* eggs fertilized with BC
- $P = P_1 + P_2$
- σ = rate at which *T. evanescens* consumes eggs of *P. Brassicae* (assuming without BC)
- ρ = rate at which *T. brassicae* consumes eggs of *P. Brassicae* (assuming without BC)
- β = Birth Rate of population
- λ = Death Rate of population

Chemical Espionage

Consider that $P_1 + P_2 = P$, where P is the total population of fertilized eggs of the Large white cabbage butterfly, *P. brassicae*. Then

$$\Delta P = \beta P_1 - (\lambda + \sigma + \rho)P_1 + \beta P_2 - (\lambda + 1.5\sigma + 1.0968\rho)P_2$$

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$$\begin{aligned}\Delta P &= \beta P_1 - (\lambda + \sigma + \rho)P_1 + \beta P_2 - (\lambda + 1.5\sigma + 1.0968\rho)P_2 \\ &= \beta(P_1 + P_2) - (\lambda + \sigma + \rho)P_1 - (\lambda + 1.5\sigma + 1.0968\rho)P_2\end{aligned}$$

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&= \beta(P_1 + P_2) - (\lambda + \sigma + \rho)P_1 - (\lambda + 1.5\sigma + 1.0968\rho)P_2 \\
&= \beta(P_1 + P_2) - \lambda(P_1 + P_2) - \sigma P_1 - (1.5\sigma)P_2 - \rho P_1 - (1.0968\rho)P_2 \\
&= \beta(P_1 + P_2) - \lambda(P_1 + P_2) - \sigma(P_1 + 1.5P_2) - \rho(P_1 + 1.0968P_2)
\end{aligned}$$

Chemical Espionage

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$$
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\Delta P &= \beta P_1 - (\lambda + \sigma + \rho)P_1 + \beta P_2 - (\lambda + 1.5\sigma + 1.0968\rho)P_2 \\
&= \beta(P_1 + P_2) - (\lambda + \sigma + \rho)P_1 - (\lambda + 1.5\sigma + 1.0968\rho)P_2 \\
&= \beta(P_1 + P_2) - \lambda(P_1 + P_2) - \sigma P_1 - (1.5\sigma)P_2 - \rho P_1 - (1.0968\rho)P_2 \\
&= \beta(P_1 + P_2) - \lambda(P_1 + P_2) - \sigma(P_1 + 1.5P_2) - \rho(P_1 + 1.0968P_2) \\
&= \beta P - \lambda P - \sigma(P_1 + 1.5P_2) - \rho(P_1 + 1.0968P_2)
\end{aligned}
$$

Chemical Espionage

The final product of the model is as follows:

$$\Delta P = \beta P - \lambda P - \sigma(P_1 + 1.5P_2) - \rho(P_1 + 1.0968P_2)$$

Considering this model, we see that our population remains relatively unchanged when

$$\beta P \approx \lambda P + \sigma(P_1 + 1.5P_2) + \rho(P_1 + 1.0968P_2)$$

From the natural equilibrium of *P. brassicae* we would have that $\beta P \approx \lambda P$ without the wasps, so to look for vulnerability we focus on the two terms $\sigma(P_1 + 1.5P_2)$ and $\rho(P_1 + 1.0968P_2)$.

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$$\text{Only } \textit{T. evanescens}: \ \sigma(P_1 + 1.5P_2)$$

The use of anti-aphrodisiac correlates to a $\approx 50\%$ increase in predation of eggs, and is a significant risk to propogation.

In this scenario, the butterfly would benefit from avoiding the anti-aphrodisiac.

$$\text{Only } \textit{T. brassicae}: \ \rho(P_1 + 1.0968P_2)$$

The use of anti-aphrodisiac correlates to a $\approx 10\%$ increase in predation of eggs, and is a lower risk to propogation

In this scenario, the butterfly can use the anti-aphrodisiac more aggressively.

$$\text{Both: } \sigma(P_1 + 1.5P_2) + \rho(P_1 + 1.0968P_2)$$

In a mixed environment, the impact would be somewhere in the middle.

Additional Problem

Problem C:

2) Suppose that the female butterfly could detect a male butterfly's propensity to use the anti-aphrodisiac prior to mating. Based on your model's results what should her strategy be in choosing a mate?

Additional Considerations

Maximal Wasps

Female *P. brassicae* will favor males not utilizing BC

Minimal Wasps

Female *P. brassicae* will have no preference for BC or non BC

Additional Considerations

i In case of minimal wasp predation, the number of males utilizing BC will not decrease any more than the original model suggested.

ii In the case of heavy wasp predation, the females would favor males without BC, which reduces the propogation of BC-utilizing males.

iii The long term eventuality of this model will be that the males using anti-aphrodisiac will be excluded from mating, and specifically their population will be compelled towards 0.

iv The fertilization rate would be presumed unchanged because the non-BC males would still be abundantly available.

Eggs of P_1

(P_1) The population of eggs created without a male utilizing BC, P_1, remains relatively unchanged.

$$\Delta P_1 = \beta P_1 - \lambda P_1 - \sigma P_1 - \rho P_1$$

Eggs of P_2

(P_2) The population of eggs created with a male utilizing BC, P_2, may change significantly based on whether the environment is a maximal wasp, or not at all in the case of a minimal wasp environment.

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- *T. evanescens* would begin to predate on *P. brassicae* at a lower rate than *T. brassicae*.
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Then

$$\mu_1 = \text{rate of decreasing predation of } \textit{T. evanescens} \text{ on } P_2.$$

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$$\mu_2 = \text{rate of increasing predation of } \textit{T. brassicae} \text{ on } P_2$$
$$\mu_1 \propto {\mu_2}^{-1}$$

Eggs of P_2 cont.

Then

$$\lim_{P_2 \to 0} (1.5\sigma - \mu_1) P_2 = 0$$

$$\lim_{P_2 \to 0} (1.0968\rho + \mu_2) P_2 = 0$$

$$\lim_{P_2 \to 0} (\beta P - \lambda P - \sigma(P_1 + 1.5P_2) - \rho(P_1 + 1.0968P_2)$$

$$= \beta P_1 - \lambda P_1 - \sigma P_1 - \rho P_1$$

Eventuality

(i.) If maximal predation is assumed, then

$$\Delta P = \beta P_1 - \lambda P_1 - \sigma P_1 - \rho P_1.$$

where $\beta = \frac{\beta_1+\beta_2}{2}$ and $\beta_2 = 0$.
(ii.) If minimal predation is assumed, the original model remains unchanged.
(iii.) If a non-static environment in which minimal and maximal wasp populations vary, the model again approaches

$$\Delta P = \beta P_1 - \lambda P_1 - \sigma P_1 - \rho P_1.$$

where $\beta = \frac{\beta_1+\beta_2}{2}$ and $\beta_2 = 0$.

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References

M. E. Huigens, J. B. Woelke, F. G. Pashalidou, T. Bukovinsky, H. M. Smid, and N. E. Fatouros, “Chemcial espionage on species-specific butterfly anti-aphrodisiacs by trichogramma wasps,” *Behavioral Ecology*, vol. 21, pp. 470–478, 2010.

M. E. Huigens, F. G. Pashalidou, M.-H. Qian, T. Bukovinszky, H. M. Smid, J. J. A. van Loon, M. Dicke, and N. E. Fatouros, “Hitch-hiking parasitic wasp learns to exploit butterfly antiaphrodisiac,” *National Academic Science USA*, vol. 106, pp. 820–825, 2009.