SCUDEM IV 2019

Problem C: Chemical Espionage

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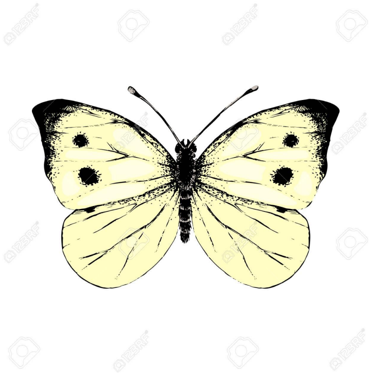
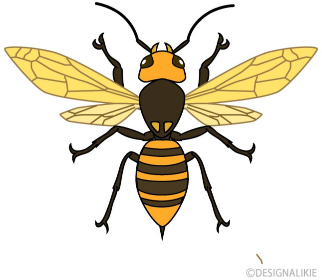
Problem

- The task was to model the relationship between the white cabbage butterflies and a species of wasp with the effect of an anti-aphrodisiac that is released by the male butterflies when they mate.
- The anti-aphrodisiacs released dissuade other males from bothering that female, but also allows the wasps to easily find the eggs and lay their own eggs inside of it.





The Basis for Our Model

- Based off of the Lotka-Volterra Predator-Prey system of ODEs
- Variables in the equations: x = Prey, y = Predator

Lotka-Volterra Model:

$$x'(t) = ax(t) - bx(t)y(t)$$

$$y'(t) = -cy(t) + dx(t)y(t)$$

Where a, b, c, d are constants

Assumptions

- Each male butterfly will mate with one female butterfly
- Anti-aphrodisiacs
  - directly proportional to the number of males mated in the population, and thus proportional to the number of butterfly eggs
  - proportional to the number of wasps
  - they positively affect both the wasps and butterfly eggs
- x = butterfly eggs
- y = wasps
- Time variable (t) is in days

Evolutions

$x'(t) = ax(t) - bx(t)y(t)$ $y'(t) = -dy(t) + ax(t)y(t)$

Constants: a = relation to anti-aphrodisiacs
b = rate of wasp larvae laid
d = death rate of wasps

We then decided to square the y(t) variable associated with the negative terms in both equations because the model was very volatile when the constants and initial conditions were slightly adjusted.

Resulting in: $x'(t) = ax(t) - bx(t)[y(t)]^2$ $y'(t) = -d[y(t)]^2 + ax(t)y(t)$

Final Model

$x(t)$ = population of Eggs - (in hundreds) t days after the start,

$y(t)$ = population of wasps (in tens) t days after the start,

a = relation to anti-aphrodisiacs

b = rate of wasp larvae layed

d = death rate of wasps

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Clear[x, y, xn, yn, apprx, a, b, c, d];
a = .1;
b = .2;
d = .2;
eqs = {x'[t] == a*x[t] - b*x[t]*y[t]^2, y'[t] == -d*y[t]^2 + a*x[t]*y[t], x[0] == 100, y[0] == 5};
apprx = NDSolve[eqs, {x, y}, {t, 0, 365}];
xn = x /. apprx[[1]];
yn = y /. apprx[[1]];
Plot[{xn[t], yn[t]}, {t, 0, 365}, PlotStyle → {Blue, Red}, PlotRange → {0, 5}]
Plot[{xn[t], yn[t]}, {t, 0, 5}, PlotStyle → {Blue, Red}, PlotRange → {0, 105}]
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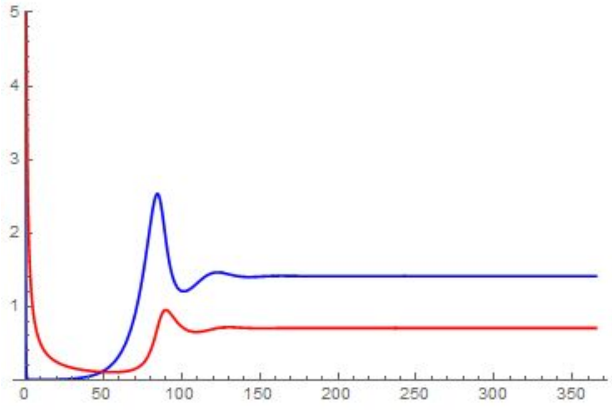
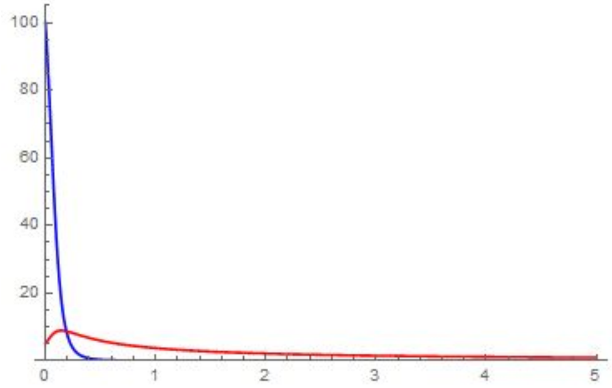
Behavior of the System

Graphs of population vs time

Note the steady state as time increases





Conclusion

In the long run the model ends up being a steady state at a set amount of wasps and butterfly eggs that sustain at a constant value.

The final steady state is only slightly dependent on the initial conditions of the eggs and wasps.

Additional Issue C-3

The effectiveness of the anti-aphrodisiacs depends on the time of day.

The constant a that accommodates the anti-aphrodisiacs in our model becomes a function of t as a sinusoidal function.

$$a(t) = (-19\cos(2\pi t) + 21)/200$$