SCUDEM 2019

Problem C

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The Problem

Determine the trade-offs and balance between the Pieris brassicae (large cabbage white butterfly) and the parasitic wasps (*Trichogramma brassicae* and *Trichogramma evanescens*). What is the best balance of this system and what is likely to happen in the long run.

Background Information

The mating of insects is done largely through chemical signals. When females release these signals, males will often release another form of chemical signal called, “ anti-aphrodisiacs”. These anti-aphrodisiacs help to mask or dissuade other males.

Chemical Espionage

Two species of wasps have been identified that can detect the anti-aphrodisiacs, and when a female butterfly has the chemical signal the wasps are more likely to follow the butterfly and lay their own eggs in the butterflies’ eggs.

Assumptions

- Perfect environment for the butterflies and wasps
- The ratio of male butterflies to female is already established and at equilibrium
- No outside interference from other predators
- The parasitic wasp will only lay eggs inside the butterflies' eggs, and therefore only exist with the presence of butterflies
- Both species of wasp act similarly enough to be considered the same

Lotka Volterra Model

Our first model began with research into predator-prey relationships. Our research brought us to the Lotka-Volterra model. This set of equations seemed promising to our goal but further development led to look for more accurate relationship modeling for the parasitic wasps and butterflies.

Lotka-Volterra Equations

$$\frac{dP}{dt} = -Pm + bHP$$

$$\frac{dH}{dt} = Hr - aHP$$

$$\begin{cases} P = P(t) & \text{Number of Predators} \\ H = H(t) & \text{Number of Prey} \end{cases}$$

$$\begin{cases} r > 0 & \text{Birth Rate of Prey} \\ m > 0 & \text{Death Rate of Predators} \\ a > 0 & \text{Death Rate of Prey/Predator} \\ b > 0 & \text{Birth Rate of Predators/Prey} \end{cases}$$

Early Modeling

Modeling of Predator vs Prey		
Predator Model	$\frac{dx}{dt} = -Ax + Bxy$	$\frac{dx}{dt} = -Ay + Bxy$
Prey Model	$\frac{dx}{dt} = Cy - Dxy$	$\frac{dx}{dt} = Cx - Dx^2 - Exy$
	Only affected by each other	**Prey is affected by some factor (i.e carrying capacity)**

First Attempt

Our first attempt at a predator-prey model was based on a purely predator-prey relationship between the butterflies and wasps with a limiting factor based on the chemical disadvantage to other male butterflies included in the prey model. After some research, we came up with two equations for relating predator and prey;

$$\frac{dx}{dt} = -0.015x + 0.00008xy$$

$$\frac{dy}{dt} = 0.2y - 0.0002y^2 - 0.006xy$$

Where x represents the population of wasps at time t, and y represents the population of the butterflies at a time t.

From this model, we found stable populations at (0,0), (27,187), (0,1000).

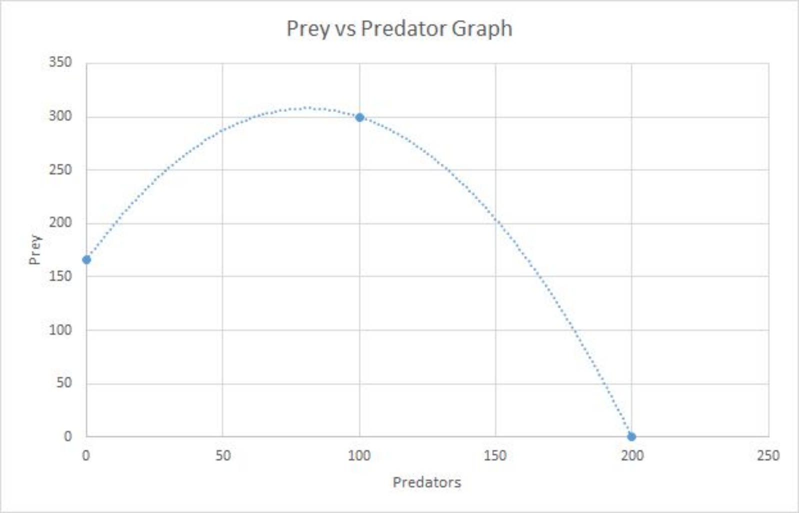
Modified Attempt

Our next attempt at modeling the relationship between the wasp and butterfly, we changed approach from a predator-prey system to a competitive system, where wasps over-infesting butterflies becomes detrimental. With this in mind, we came up with the following:

$$\frac{dx}{dt} = 0.6x - 0.003x^2 - 0.001xy$$

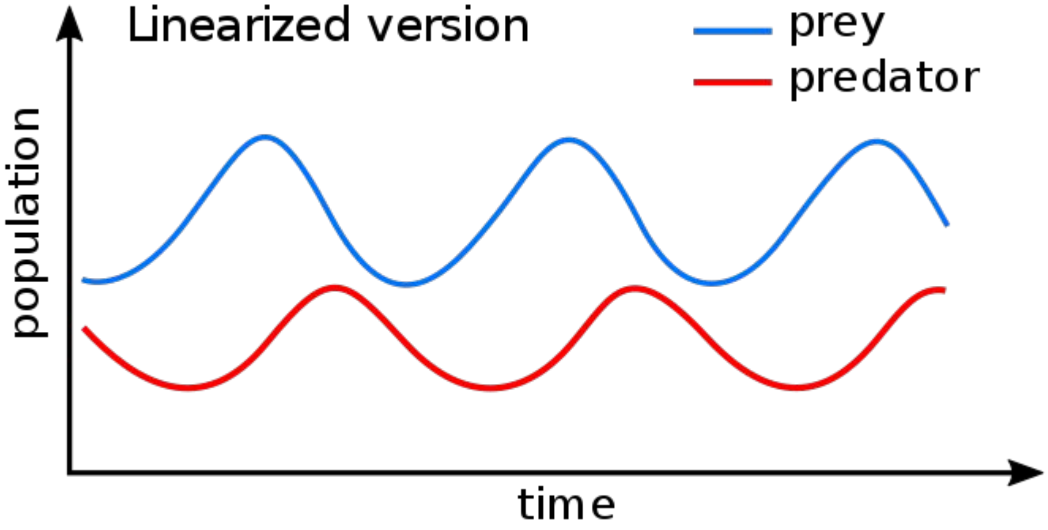
$$\frac{dy}{dt} = 0.3y - 0.002y^2 - 0.003xy$$

From this model, when we solve for stability, we find stable populations at (0,0), (100,300), (0,166).



Long Term Observation

Due the assumptions and factors we considered the populations of both species would fluctuate over time but still maintain a pattern as predator or prey increases or decreases



Problems We Encountered

At first, we attempted to tackle every aspect of the problem at once and ended up overcomplicating it. Moving forward, we realized it was easier to start with a simpler, more broad model and expand it to consider each aspect of the overall problem.

A large drawback was lack of knowledge in the given field. We decided to challenge ourselves and choose a problem from a field we are not familiar with, and so most of our time was spent on research. Additionally, we have not yet finished Differential Equations as a class and do not have the optimal background to tackle our problem.

Moving Forward

Given more time and preparation, we would conduct more thorough and meaningful research on predator-prey relationships, parasite-host relationships, and differential equations modeling population growth. With more knowledge and experience, we could develop a more complete and adaptable model for the situation and each of its constraints.

Questions?