YOUNGSTOWN STATE UNIVERSITY

Chemical Espionage

Yogesh Sapkota, Pradip Rimal, Bishal Lamichhane

Youngstown State University

November 10, 2019

Advisor: Dr. Eric Wingler

Pieris brassicae

- Cabbage white butterfly
- Lays 40-100 eggs in cluster

Figure: A Large White Female *Pieris brassicae*

Anti-aphrodisiacs

- Transferred by male *Pieris brassicae* during mating
- Repels other male *Pieris brassicae*
- Arrests parasitic wasps like T. brassicae wasps three days after butterfly egg deposition[1].
- The larvae of parasitoid wasps feed on the eggs laid by female *Pieris brassicae*
- Male *Pieris brassicae* harasses female *Pieris brassicae* during oviposition

The problem

- Two competing interests
- Model the interaction between the parasitoids and host(*Pieris brassicae*)
- Find the best balance for this interactions

Figure: Trichogramma wasp on *Pieris brassicae* eggs

Assumptions

- The model considers a population of a single colony of Pieris brassicae for analysis
- Every wasp that clings to the mated female moth is assumed to successfully destroy all the eggs she lays.
- Intra-specific competition amongst the wasps is not taken into consideration.
- Both healthy and infected moths are considered equally likely to be attaked by the wasps.
- We haven't considered the age-structure, i.e. the parasitoids do not discriminate on the basis of the age

Parameters

- B(t)= population of *Pieris brassicae* at time t,
- W(t)= population of parasitic wasps at time t,
- r= intrinsic growth rate of *Pieris brassicae*,
- B_f = Population of mated female *Pieris brassicae*,
- B_m = Population of male *Pieris brassicae*,
- α_1= number of eggs laid by one female *Pieris brassicae*
- α_2= proportion of α_1 impacted by the harassment of male *Pieris brassicae*
- d = death rate of the parasitic wasps,
- γ= conversion efficiency, number of parasitoids that emerge from parasitized *Pieris brassicae*
- $f(B_f)$ and $g(B_f)$ are functional responses.

The base Model

- Without the interaction

$$\begin{aligned} \frac{dB}{dt} &= rB \\ \frac{dW}{dt} &= -dW \end{aligned} \tag{1}$$

- With the interaction between hosts and parasitoids but without host feeding

$$\begin{aligned} \frac{dB}{dt} &= rB - f(B_f)W \\ \frac{dW}{dt} &= -dW + f(B_f)\gamma W \end{aligned} \tag{2}$$

The model

- With host feeding

$$\begin{aligned} \frac{dB}{dt} &= rB - f(B_f)\alpha_1 W \\ \frac{dW}{dt} &= -dW + f(B_f)\gamma W \end{aligned} \tag{3}$$

- With host feeding and male interference

$$\begin{aligned} \frac{dB}{dt} &= rB - f(B_f)\alpha_1 W - g(B_f)B_m\alpha_2 \\ \frac{dW}{dt} &= -dW + f(B_f)\gamma W \end{aligned} \tag{4}$$

Functional Response

- Functional response: Relationship between the number of prey or hosts attacked by a predator or parasitoid as a function of prey density[1],[2].
- Can be extended to parasitoids.
- Type II functional used for *Trichogramma brassicae* .
- Using the famous disc model for predators[1], we get

$$f(N) = \frac{aN}{1 + aT_hN} \tag{5}$$

where N is the number of prey, a the encounter rate while searching, and T. the handling time.

- f(N) gives the number of encounters with hosts per parasitoid perunit time for our model.

1 2

[1] Some Characteristics of Simple Types of Predation and Parasitism, Hollling

[2] The Natural Control of Animal Populations,M. E. Solomon

Functional Response for Our Model

- function of B_f
- influenced by the concentration of anti-aphrodisiacs produced.
- $g(B_f)$: functional response for the interaction between male *Pieris brassicae* and mated female *Pieris brassicae* and
- $f(B_f)$: functional response for the the interaction between mated female *Pieris brassicae* and the parasitoids

$$f(B_f) = \frac{kB_f}{1 + khB_f} \tag{6}$$

$$g(B_f) = \frac{\beta B_f}{1 + \beta h_1 B_f} \tag{7}$$

Functional Response for Our Model

$$f(B_f) = \frac{kB_f}{1 + khB_f}$$

$$g(B_f) = \frac{\beta B_f}{1 + \beta h_1 B_f}$$

- k = the encounter rate of parasitoid with the mated female,
- β= is the encounter rate of male with the mated female,
- h = handling time for the parasitoids,
- h_1= handing time for the males.

The Concentration

- Anti-aphrodisiacs attract the parasitoids(the k factor) and repel males(the β factor).
- Assume k and β depend upon the concentration of anti-aphrodisiacs
- We assume the following relationship between k, β and c, factor incorporating the concentration of anti-aphrodisiacs: $k = \mu_1 c$, $\beta = \frac{\mu_2}{c}$, where μ_1 and μ_2 are two constants.

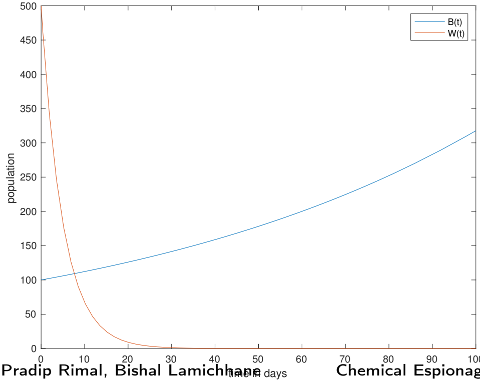
The Final Model

$$\begin{aligned}\frac{dB}{dt} &= rB - \frac{\mu_1 c B_f}{1+\mu_1 c h B_f}\alpha_1 W - \frac{\mu_2 B_f}{c+\mu_2 h_1 B_f} B_m \alpha_2 \\ \frac{dW}{dt} &= -dW + \frac{\mu_1 c B_f}{1+\mu_1 c h B_f}\gamma W\end{aligned} \tag{8}$$

Extreme Values

- When $c \to 0$ the equations take the form;

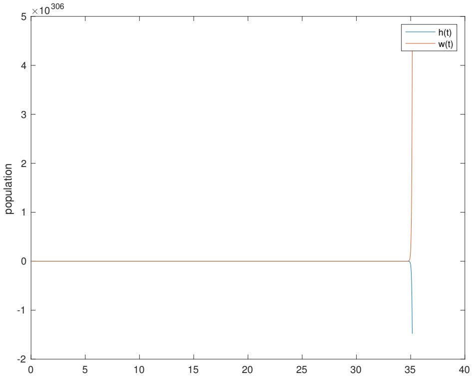
$$\begin{aligned} \frac{dB}{dt} &= rB - \frac{\alpha_2}{2h_1}B \\ \frac{dW}{dt} &= -dW \end{aligned} \tag{9}$$



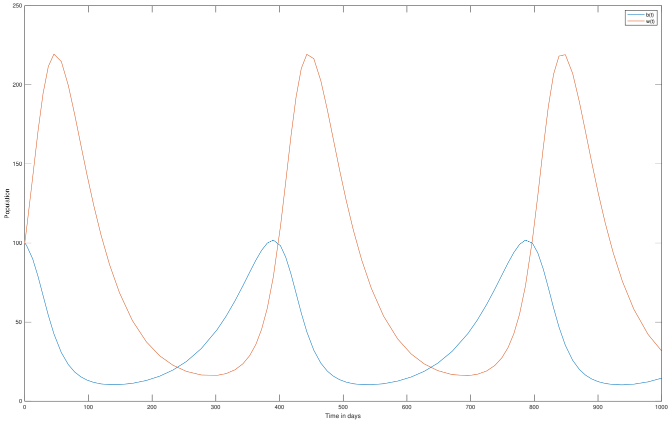
Extreme Values

- When $c \to \infty$ the equations take the form;

$$\begin{aligned} \frac{dB}{dt} &= rB - \frac{\alpha_1}{h}W \\ \frac{dW}{dt} &= -dW + \frac{\gamma}{h}W \end{aligned} \tag{10}$$



- when c= 0.00001;



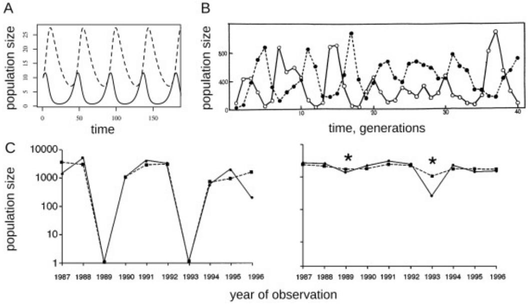


Figure: A:Lotka–Volterra-type, interconnected population cycles of host (solid line) and parasite (dashed line). (B) Cyclical population size changes of azuki bean weevil (solid line, open circles) and its parasitoid wasp (dashed line, filled circles) in the Utida experiment[3]

3

[3] Host–parasite coevolution: why changing population size matters

Results

- The impact due to interference of male *Pieris brassicae* is very low compared to the impact due to the host feeding by the parasitoids.
- In the favor of female *Pieris brassicae*, it's advantageous to attract more male than the parasitoids.

Future Improvements

- Utilize a more sophisticated relation between c, k and β,
- Incorporate the age-structure model
- Consider Intra-specific competition among the parasitoids

Additional Issue

Issue: 3 A female should choose a mate that releases anti-aphrodisiac with concentration corresponding to $c \approx 0.00001$ unit per female per day.

Issue: 1

$$c = \frac{c_o}{2}\left[1 + \sin(2\pi(t - \frac{1}{4}))\right]$$

- When $t = 0$ (Midnight), $c = 0$
- When $t = \frac{1}{2}$ (Noon), $c = c_o$

Functional response:

$$f(B_f) = \frac{\mu_1 \frac{c_o}{2}\left[1 + \sin(2\pi(t - \frac{1}{4}))\right] B_f}{1 + \mu_1 \frac{c_o}{2}\left[1 + \sin(2\pi(t - \frac{1}{4}))\right] h B_f}$$

$$g(B_f) = \frac{\mu_2 B_f}{\frac{c_o}{2}\left[1 + \sin(2\pi(t - \frac{1}{4}))\right] + \mu_2 h_1 B_f}$$

Additional Issue

Taking $c_0 = 0.00001$;

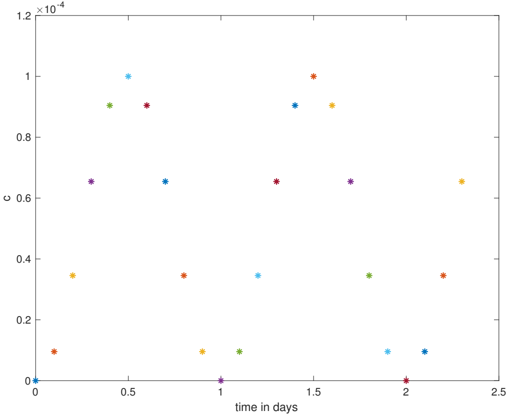


Figure: Change in c with respect to the time of the day

Additional Issue

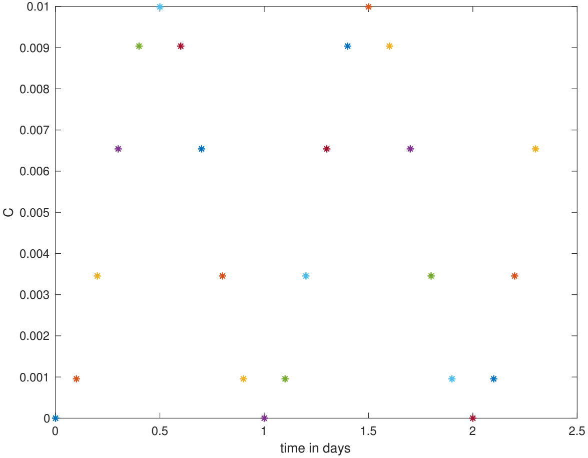


Figure: Change in $f(B_f)$ with respect to the time of the day

Additional Issue

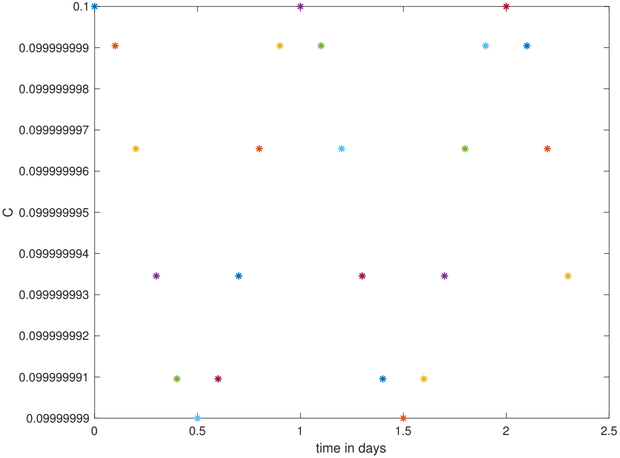


Figure: Change in $g(B_f)$ with respect to the time of the day

Thank You.

Yogesh Sapkota, Pradip Rimal, Bishal Lamichhane