Problem A - Fashion Trends: How They Come and Go

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Description /Assumptions about the System being Modeled

We are modeling the flow of a population through a fashion trend, specifically types of shoes. Our model focuses on a main type of shoe (B) which follows an initial shoe trend (A) and comes before a final shoe trend (C). In this model we are assuming no one can move back to a previous trend. The phenomenon of a trend returning is not considered here, as we are not considering a large period of time usually necessary for a trend to return to popularity.

Each of our populations in the three different trends are broken into two groups, non-conformists and conformists, giving us 6 total groups. A non-conformist is motivated to change to the next trend by a high influence of his current trend relative to the influence of the next trend. This models an individual who wishes to be unique. Non-conformists are important to our model as they start the growth of the next trend. Conformists are motivated to change to the next trend by a high influence of said trend relative to their own trend. All these groups also have their own influence on the total population.

The influence of all 6 groups will be proportional to the total number of connections individuals in specific groups have to other individuals in the entire population. We define a connection to be relationship where one individual makes known to the other what trend he is a part of, either through social media or real-world interactions. We assume that an individual has the same amount of influence over all other individuals, regardless of what shoe they wear. We are also assuming that an individual's chance of changing to the next shoe trend is purely a function of what influences them and not the influence they have. Influence is an important part of our model as we can create realistic initial conditions where a smaller group is in the second trend but have a disproportionate amount of influence on the rest of the population. This will in affect "kick-start" the growth of the second trend.

Parameters and Variables

First, we have the variables that describe the size of the populations of each six groups. The population of conformists in group A will be called A_+, and non-conformists in population A will be called A_-. B_+, B_-, C_+, and C_- are defined similarly. Next, we have variables to describe the influence each group has on the entire population. The influence of group A_+ will be called I_{A+}. I_{A-}, I_{B+}, I_{B-}, I_{C+}, and I_{C-} are defined similarly. Note that the ratio of the influence variables is what affects our model, because it shows how much more likely an individual is to choose one shoe over another.

If there existed no flow out of shoe C then the entire population would build up there, which would greatly skew the results. To combat this error, we assume an amount of people proportional to the population that wears shoe C are moving on to the next shoe trend. We define this proportionality coefficient as $n \in [0,1]$ (for our calculations we assumed a relatively small $n = 0.3$ as this produced similar behaviors in C and B). We also add two variables k_1 and k_2 to represent our conformist and non-conformist flow rate. Without k, our model would simulate that 100% influence from a single group would make 100% of individuals change (or stay for non-conformists) in a single unit of time. Since this might not be true, we introduce k_1 and k_2 both $\in [0,1]$ to vary this proportion.

System of Equations

In our system we must describe the flow of conformists, non-conformists, conformist's influence, and non-conformists influence for each group A, B, and C. For the flow of conformists from A_+ to B_+, we must consider the ratio of influence from B on A_+ to all influence from A and B on A_+ which equals $I_B/(I_B+I_A)$. This value represents how much a conformist wants to move to population B, because the higher the influence B has the more they wish to change to it. This multiplied by our population A will be proportional to the flow and we multiply by our conformist flow rates to get the actual flow; thus, we arrive to the formula:

$$\frac{dA_+}{dt} = -\frac{I_B}{I_A + I_B} \cdot A_+ \cdot k_1$$

To get the flow of influence from A_+ to B_+. We simply multiply the flow of A_+ by the average influence of each member in A_+ which equals the influence divided by the population. Thus, we have:

$$\frac{dI_{A+}}{dt} = \frac{dA_+}{dt} \cdot \frac{I_{A+}}{A_+}$$

To get the flow of from A_- to B_- we follow a similar model except non-conformists are more motivated to leave if the influence of their own group is high relative to the total influence from A and B. We also must multiply our final flow by the non-conformist flow rate. Thus, for A_- and I_{A-} we have:

$$\frac{dA_-}{dt} = -\frac{I_A}{I_A+I_B} \cdot A_- \cdot k_2 \qquad \frac{dI_{A-}}{dt} = \frac{dA_-}{dt} \cdot \frac{I_{A-}}{A_-}$$

For the four different rates of flow for group B we do the same except we must account for the flow into B from group A. Thus, the equations describing the flow of population are:

$$\frac{dB_+}{dt} = -\frac{dA_+}{dt} - \frac{I_C}{I_B+I_C} \cdot B_+ \cdot k_1 \qquad \frac{dB_-}{dt} = -\frac{dA_-}{dt} - \frac{I_B}{I_B+I_C} \cdot B_- \cdot k_2$$

$$\frac{dI_{B+}}{dt} = -\frac{dI_{A+}}{dt} - \frac{I_C \cdot I_{B+}}{I_B+I_C} \cdot k_1 \qquad \frac{dI_{B-}}{dt} = -\frac{dI_{A-}}{dt} - \frac{I_B \cdot I_{B-}}{I_B+I_C} \cdot k_2$$

For C we have the inflows from B and also our outflows which we assume are proportional to the population of C for some small proportionality constant. Thus, we have:

$$\frac{dC_+}{dt} = \frac{I_C}{I_B + I_C} \cdot B_+ \cdot k_1 - n \cdot C_+ \qquad \frac{dC_-}{dt} = \frac{I_B}{I_B + I_C} \cdot B_- \cdot k_2 - n \cdot C_-$$

For influence we multiply each flow by their respective average influence to get.

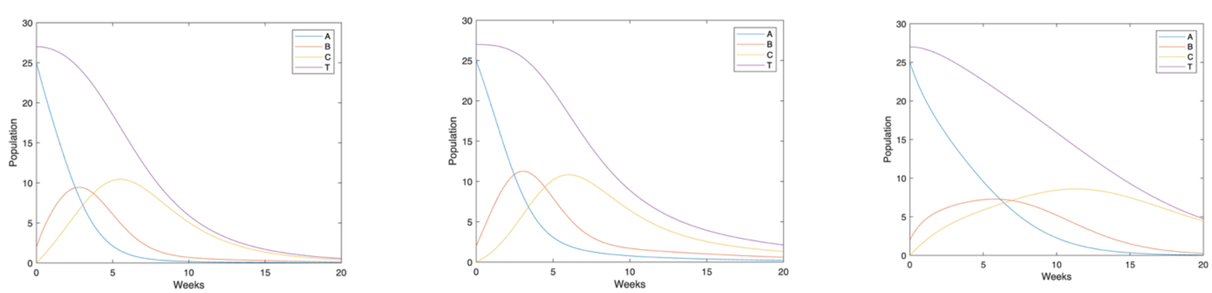
$$\frac{dI_{C+}}{dt} = \frac{I_C \cdot I_{B+}}{I_B + I_C} \cdot k_1 - n \cdot I_{C+} \qquad \frac{dI_{C-}}{dt} = \frac{I_B \cdot I_{B-}}{I_B + I_C} \cdot k_2 - n \cdot I_{C-}$$

Results

Our first initial conditions model a disproportionately influential group of non-conformists partaking in another trend, and due to that high influence, conformists begin to change to said trend, so initial conditions will be:

$$A_+ = 20, A_- = 5, B_+ = 0, B_- = 2, C_+ = 0, C_- = 0, I_{A+} = 20, I_{A-} = 5, I_{B+} = 0, I_{B-} = 5, I_{C+} = 0, I_{C-} = 0,$$

We will also look at cases where $k_1 = k_2 = 1$ (below left), where $k_1 = 1$, $k_2 = 0.5$ (below middle), and where $k_1 = 0.5$, $k_2 = 1$ (below right). Also, we are letting T be the total population in the three groups.



Trend B reaches its maximum of 9.5 at $t = 2.8$ for the first graph, 11.2 at $t = 3$ for the second, and 7 at $t = 7$. We see that the easier conformist change relative to non-conformists translates to a faster and larger growth of trend B.