Problem A: Group Affinity and Fashion Sense: Color-Wheel Model

Introduction. What exactly determines the propensity for a person to alter their appearance? This paper will focus on how people identify the mainstream trend, and how different people are influenced by each other to either conform to that trend or move away from it. We modeled a group of people's reactions to style trends and the overall dynamic of the style choices using what we name the "Color-Wheel Model." Similar to Jonathon Touboul's model, Our model considers two types of people within a population: non-conformists ("hipsters"), who move away from what they identify as the mainstream trend, and conformists, who adhere to current societal trends (Touboul, 2014).

We considered several factors that influence a person to change their style, such as how similar their style is to the mainstream trend and how "extreme" or unique their style is, and the extent to which interactions influence a person to change styles.

Assumptions.
1. The model exists in a closed system, where the number of total population (n) is constant.
2. The number of hipsters and the number of conformists is constant.
3. Each individual identifies trend in the same way, by observing the style choices other people.
4. Each individual acts as soon as they are convinced to change their style (i.e. there is no delay).
5. Different kinds of interaction which include interaction through social media, face-to-face encounters, and even indirect influence by celebrities, are compacted into one variable.

Model. We set our domain as $x^2 + y^2 \leq 1$. Each $\vec{p_i}$ within the domain is the position vector representing the choice of style by person i in the 2-D plane of different possibilities of styles. Compare this to a color wheel, whereby each color represents a style. Therefore, the number of different styles that an individual can assume is infinite.

• Style Change. The movement of $\vec{p_i}(t)$, denoted by the directional vector $d\vec{p_i}(t)/dt$, represents the change person i makes to the style.

• Trend. For our population of size n, define $\vec{T} := \frac{1}{P}\sum_{i=1}^{n} \vec{p_i}$ where $\vec{T}$ represents the general trend.

• Influence by Interaction. $(\alpha_i * \beta_i)$ represents the extent to which person i is influenced by other people's style choices, where α_i is how easily they are convinced by interactions to change their style. β_i represents the amount of interaction by person. i $(0 < \alpha_i < 1)$

Given above considerations, the behaviors of people following the identification of trend is modeled:

If person i is a hipster,

$$\frac{d\vec{p_i}}{dt} = -\frac{(\vec{T} - \vec{p_i})}{\mid \vec{T} - \vec{p_i} \mid} f(\mid \vec{T} - \vec{p_i} \mid)(1- \mid \vec{p_i} \mid)\alpha_i\beta_i k_h \qquad , k_h = \text{constant} \tag{1}$$

If person i is a conformist,

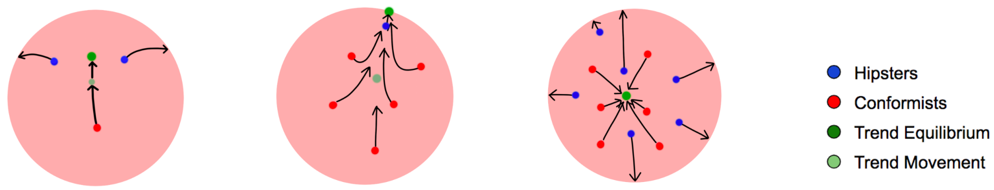
$$\frac{d\vec{p_i}}{dt} = \frac{(\vec{T} - \vec{p_i})}{\mid \vec{T} - \vec{p_i} \mid} g(\mid \vec{T} - \vec{p_i} \mid)\alpha_i\beta_i k_c \qquad , k_c = \text{constant} \tag{2}$$

I. $\pm\frac{(\vec{T}-\vec{p_i})}{|\vec{T}-\vec{p_i}|}$ is a unit vector representing the direction of movement of $\vec{p_i}$, toward or away from $\vec{T}$.

II. $f(\mid \vec{T} - \vec{p_i} \mid)$, $g(\mid \vec{T} - \vec{p_i} \mid)$ is the speed of movement as a function of the extent of difference between trend and current style choice. (ie. $f(\mid \vec{T} - \vec{p_i} \mid) = 2- \mid \vec{T} - \vec{p_i} \mid$; $g(\mid \vec{T} - \vec{p_i} \mid) =\mid \vec{T} - \vec{p_i} \mid)$

III. $(1- \mid \vec{p_i} \mid)$ ensures that $\vec{p_i}$ stays inside $x^2+y^2 \leq 1$, as $\lim_{\vec{p_i}\to 1}(1- \mid \vec{p_i} \mid) = 0$. If $\mid \vec{p_i} \mid$ exceeds 1, then the direction of $d\vec{p_i}/dt$ reverses as $(1- \mid \vec{p_i} \mid)$ becomes negative. For conformists, $\mid \vec{p_i} \mid \leq 1$ always.

Analysis. We analyzed the long-term behavior of the model with different numbers of conformists and hipsters. When there are two hipsters, they will both move in the opposite directions from each other until they reach the boundary, and will remain there. When there are two conformists, they will move toward each other and converge at a midpoint. In this situation, $\forall i$, $\vec{p_i} = \vec{T}$. When there is a conformist and a hipster, the hipster will move in the opposite direction from the conformist and the conformist will move toward the hipster. **In the long-term, the hipster and conformist will end up in the same position on the boundary.**



(a) One conformist. (b) One hipster. (c) Hipsters and conformists.

Figure 1: Long-term behavior.

Now consider situations where there is only one hipster or one conformist within a population of size greater than 2. From Figure 1(b), if there are multiple conformists and one hipster, conformists converge to $\vec{T}$, which slowly approaches the position of the hipster. Eventually, all $\vec{p_i}$'s converge to one point on the boundary of the domain. This behavior illustrates that when people perceive an individual as a hipster (or a trend-setter), they slowly mimic the style choice of the hipster and everyone ends up sharing the same style, in particular, the $\vec{p_i}$ on the boundary. Therefore, we say conformists identify hipsters to be the trend-setters, therefore are inclined to follow some aspects of hipsters' style-choices.

If there are multiple hipsters and conformists, the hipsters eventually land at different points along the boundary, whereas all conformists converge to a point within the domain which is equal to the position of $\vec{T}$, and is also the center of gravity of $\vec{p_i}$'s for all hipsters, $\vec{H} = \sum \vec{p_{hi}} = \vec{T}$. See Figure 1(c).

The extent of similarity of styles among hipsters and conformists is shown by the distance between their positions. If they converge to a point, it means their styles have a high level of similarity, or they have low level of similarity. The model's long-term behavior does not change when we change the scalar terms, it will only change the speed of convergence, and maybe shift the equilibrium position of $\vec{T}$.

Conclusion. In this paper, we have studied the behavior of a model that simulates the decisions made by either hipsters or conformists to change their appearances based on other people's appearances. Given that conformists follow trends and hipsters ignore trends, we found that, long-term, conformists always converge to the mainstream trend and eventually share the same style, and hipsters always tend toward the boundary. In addition, conformists follow the style choices of hipsters. Eventually, the trend of the total population, will equal to the trend of the hipsters alone.

Further Improvements. We have noticed that when there are multiple conformists and one hipster (see Figure 1(b)), the hipster eventually gets cornered by a group of conformists toward a point on the boundary. In this case, the hipster stops moving, as $1- \mid \vec{p_i} \mid$ becomes 0. To prevent this situation, the model can be further developed to have hipster had a wider range of movement instead of moving in the complete opposite direction from $\vec{T}$.

Finally, the interaction factor, which we assumed to be constant for simplicity, is worthy of being examined in closer detail. For example, does Twitter cause more makeovers than face-to-face conversations do? Is it the other way around? Earlier, we attempted to incorporate this seeming interdependence in the current model. Let $f_i = \alpha_{f_i}\beta_{f_i}k_{m_i}\beta_{m_i}$, where α_{f_i} is the probability that a face-to-face interaction convinces person i, and β_{f_i} is the number of these interactions they have. k_{m_i} measures what percentage of person i's social media experiences will affect their offline interactions, while β_{m_i} measures the number of social media interaction that they have. Define m_i similarly with m_i indices switched with f_i.

References

[1] Touboul, Jonathon, “The Hipster Effect: When Anticonformists All Look The Same,” https://arxiv.org/abs/1410.8001 . Accessed October 2019