Problem A: Group Affinity and Fashion Sense
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(1) Problem Statement

The goal is to develop a model that will describe people in an established group and how they interact and decide to change their appearance. The model is to clearly describe the aspect being changed, describe how information about the idea is exchanged between people within the established group, describe interactions between people in the group, and describe the range of values for parameters and the meaning of higher and lower parameters.

(2) Model Description

For our model, we focus on an established group of people interacting through Twitter, a social networking site where users can post original content, retweet and like original content from other users, and reply and directly message other users. These users can alter their "appearance" primarily through the content that they retweet and like. Thus, we arbitrarily define someone's appearance on Twitter as the content someone would view if they clicked on that individual's profile (i.e. their retweets, likes, and personal original content). We assume that, within this established group, the only content that they view is from the other members of the established group (i.e. they only follow those within their established group). Additionally, once an individual adopts an idea, they begin to change their appearance.

In our model, there exists four groups: susceptible (S), infected (I), resistant (R), and anti (A). That is, for a specific idea in an established group, one can become *infected* (exposed to the idea and becomes an advocate), *resistant* (exposed to the idea and does not feel passionately enough about the issue to develop an opinion), *anti* (exposed to the idea and becomes a critic), and *susceptible* (not yet exposed to the idea or has not yet made a decision). These populations add in a manner that their total is equal to the total population of the established group ($S + I + R + A = N$). The change in total population is assumed to be zero.

Within this established group, we define two types of interactions: **passive** and **active**. Passive interactions include viewing content created and shared by others, while active interactions are replies and direct messages.

Passive interactions have contributions from both the infected group and the anti group, while the resistant group is assumed to not be contributing relevant content. We define τ as the content viewed by an individual that is in favor of an idea. This content is created at a rate of Φ (with units of time^{-1}). Similarly, we define β as the content viewed by an individual that is produced by the anti group at a rate of κ (also with units of time^{-1}). Viewing content produced by either group will push an individual to that group. Finally, to contribute to the resistant group, we define γ to be a parameter that accounts for a susceptible group member being exposed to an issue and not feeling passionate about it. Individuals are more comfortable not choosing a side when there are other individuals who have become resistant themselves or have not yet decided.

$$\tau(I) = I\phi$$

$$\beta(A) = A\kappa$$

$$\gamma = \frac{(\phi + \kappa)S}{N}$$

Next, we account for the active interactions. ψ is defined as the amount of interactions that an individual experiences. We make the assumption that all interactions are similar, but the way the individual reacts to them varies and is related to their initial opinion. We quantify this difference in reactions through the parameter a. The rate at which interactions occur is determined by the coefficient ξ (with units of time^{-1}).

$$\psi\,(I, A) = \xi(I + A)$$

(3) Final Model and Evaluations

Accounting for all of these factors, we arrive at the final set of equations for our model:

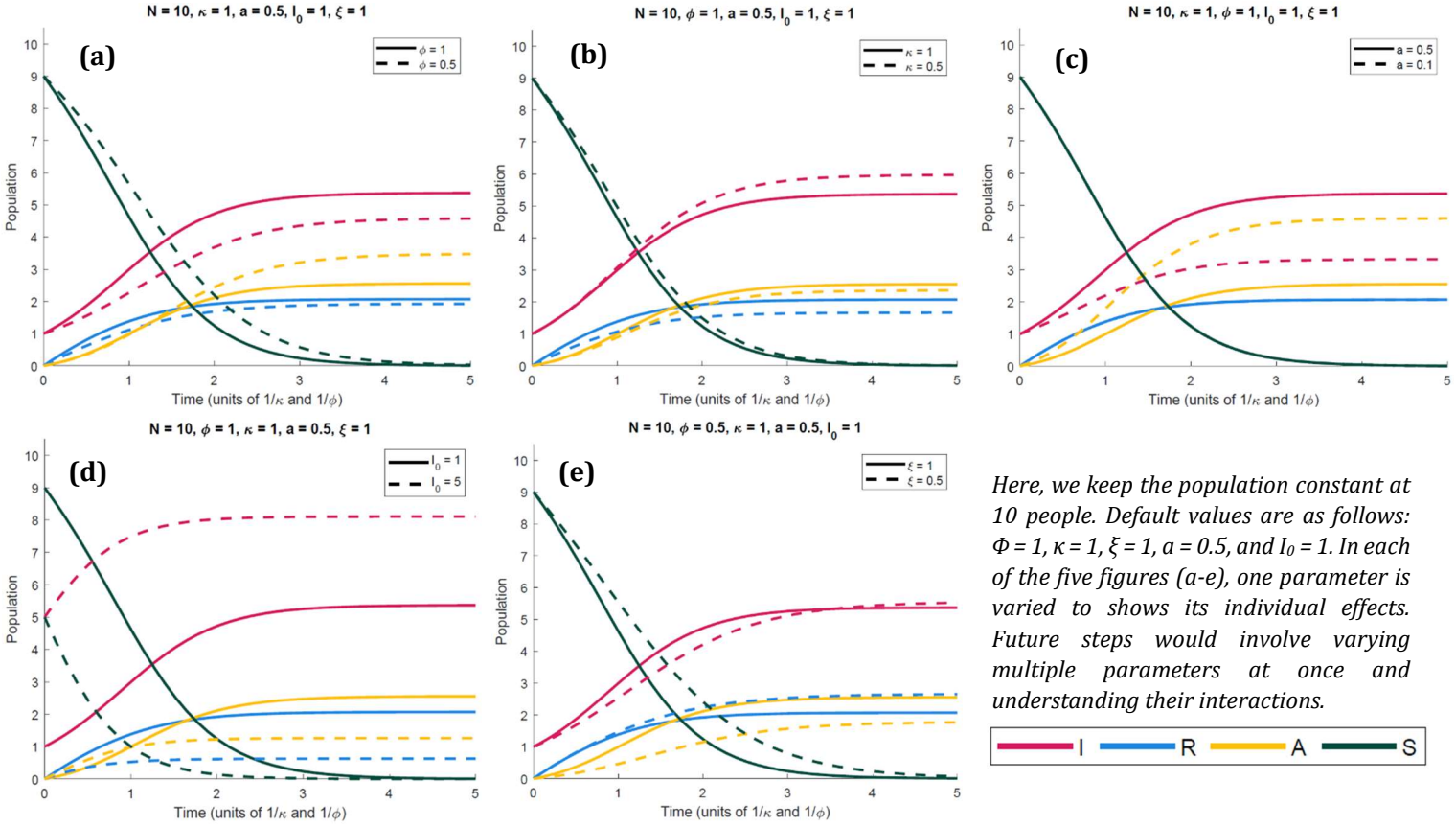
$$\frac{dS}{dt} = -\frac{\tau S}{N} - \frac{\beta A}{N} - \gamma S - \frac{S\psi}{N}$$

$$\frac{dI}{dt} = \frac{\tau S}{N} + a\left(\frac{S\psi}{N}\right)$$

$$\frac{dR}{dt} = \gamma S$$

$$\frac{dA}{dt} = (1-a)\left(\frac{S\psi}{N}\right) + \frac{\beta S}{N}$$

We now vary each of our independent parameters (Φ, κ, ξ, a) as well as I_0.



Here, we keep the population constant at 10 people. Default values are as follows: Φ = 1, κ = 1, ξ = 1, a = 0.5, and I_0 = 1. In each of the five figures (a-e), one parameter is varied to shows its individual effects. Future steps would involve varying multiple parameters at once and understanding their interactions.

The results of varying parameters matches our intuition (e.g. increasing Φ, which represents the rate at which infected individuals produce content, results in an increase in infected individuals in the final population).

As mentioned above, once an individual becomes infected (or joins the anti group), they will begin to change their appearance (V). This change in appearance would manifest itself in changes in likes (ρ), retweets (σ), and original content (χ). Further improvements of this model would account for these changes in available content (C).

$$\frac{dV}{dt} = \frac{d\rho}{dt} + \frac{d\sigma}{dt} + \frac{d\chi}{dt} \qquad \frac{dC}{dt} = \sum_{n=1}^{n=N} \frac{d\rho_n}{dt} + \frac{d\sigma_n}{dt} + \frac{d\chi_n}{dt}$$