Executive Summary: Problem A- Group Affinity and Fashion Sense

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Problem Statement: This modeling problem is about the social dynamics (interactions of individuals, change of trends etc.) within and between two opposing groups: *hipsters* and *mainstreams* in a society, and how the transition and information exchange occur between the two groups.

I. INTRODUCTION

In our approach, we have considered fashion sense, which differs among individuals and groups, to be the changing part of appearance. Based on that, individuals can roughly be categorized into two distinct subgroups, *hipsters* and *mainstreams*. *Hipsters*, by definition, try to do what is not common. So, when it comes to fashion, they try to dress uniquely. This is in sharp contrast with the *mainstreams* who follow the common current. As a new trend kicks in, waves of information reach people through interpersonal connections. Our model simulates the social behaviors among people (within and outside their subgroups) and social environmental factors to predict how individuals change their appearance and mentality with time.

II. ASSUMPTIONS

1) Based on Dunbar number[1], we assume that there is a limit in the number of *meaningful* social relationships an individual can have.
2) Population of a society doesn't change during the simulation period.
3) If two people have never met and neither of them are famous (someone with an extremely high outward bonds relative to the average person), they cannot share any bonds.
4) Relationships grow and decay in the same direction on both sides.

III. MODELING*

A. Modeling of Social Structure

We use graphs, consisting of nodes and edges, to represent social structures as follows:
Nodes represent individuals, which consist of individuals' three inherent traits (all the values are continuous and they range between $0 - 1$):

1) Hipster index (h): represents *hipsterness* of a person; a person with higher h value is more likely to be a *hipster* whereas one with lower h value is more likely to be a *mainstream*,
2) Influencing index (ϕ): represents a person's ability to influence others; higher ϕ value means a person has higher influencing power, and
3) Responsiveness index (ρ): represents a person's responsiveness to other people's influence; higher ρ value means a person is less likely to be influenced by other people.

Weighted (bi-directional) edges denote bonds between individuals. The weights on the directed edges between individuals i and j range

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*The unexplained constants presented in this paper were determined experimentally.

between $0-1$ and represent the strength of the bond between them in the following way:

- b_{ij}: strength of the relationship as perceived by j; higher b_{ij} value means the person j values the bond more and vice versa,
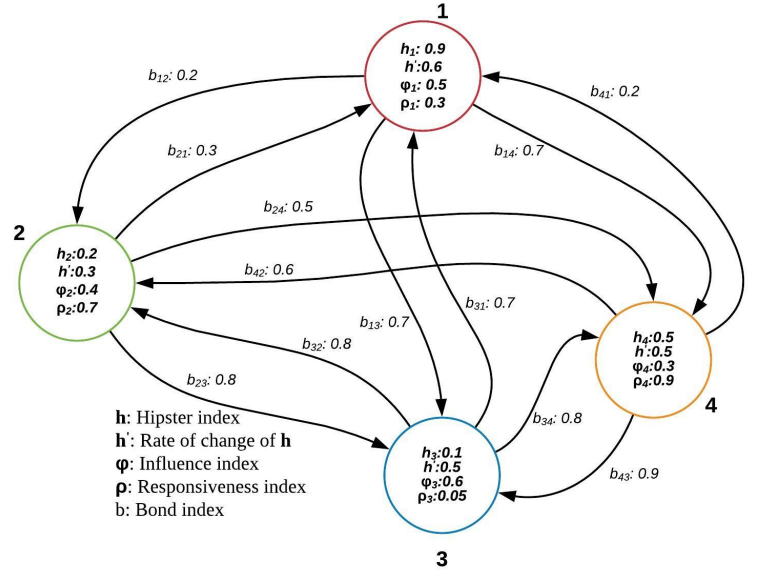- b_{ji}: strength of the relationship as perceived by i.

Fig. 1: A social structure of four individuals, their h-values and corresponding parameters

The social structure at time t can also be represented as a matrix that we will call Bond Matrix, $B(t)$. The corresponding Bond matrix of the social structure represented in Fig. 1 at $t = 0$ is $B(0)$.

$$
B(0) = \begin{bmatrix} 0.0 & 0.2 & 0.7 & 0.7 \\ 0.3 & 0.0 & 0.8 & 0.5 \\ 0.7 & 0.8 & 0.0 & 0.8 \\ 0.2 & 0.6 & 0.9 & 0.0 \end{bmatrix} \tag{1}
$$

B. Modeling of Social Dynamics

To model the dynamics in social connections, we introduce two types of Randomness Matrices: *External Randomness Matrix* and *Self-induced Randomness Matrix*.

External Randomness Matrix (R_{ex}) mimics the changes of interpersonal bonds caused by social interactions. First time meetings and the elimination of relationships are accounted for. R_{ex} is a square matrix with zeros on the main diagonal, and random numbers ($r[ij]$) in remaining entries. Features of R_{ex} are below:

- All entries follow a normal distribution centered at 0 with standard deviation 0.2
- $r[ij] \cdot r[ji] \geq 0$ (by assumption 4)

At periodic time intervals, R_{ex} is added to the Bond Matrix (B) to give the randomized Bond Matrix, $B_r(t)$:

$$B_r(t) = B(t) + R_{ex}(t) \qquad (2)$$

Self-induced Randomness Matrix (R_{si}) mimics the changes in the individuals caused by themselves by reading, researching etc. R_{is} is a column matrix of random numbers. The numbers are generated in such a way that non-zero numbers have occurrence probability of 0.2. Also, the non-zero numbers follow normal distribution centered at 0 with standard deviation 0.2. During the same periodic time interval, the Bond Matrix is randomized (R_{si}) and is added to the the Hipster index matrix, H, to generate randomized Hipster Index Matrix, H_r.

$$H_r(t) = H(t) + R_{si}(t) \qquad (3)$$

C. Differential Equation Construction

We include three terms in our second order differential equation, Social Influence, Filter Bubble[2] Drag, and a Bimodal Potential Well[3]. The main contributor to the dynamics of the hipster value is the Social Influence, which for a person is comprised of the sum of influences from every person bonded to that individual, incorporating the social parameters of responsiveness, influence, and bond strength.

$$C_0 \rho_i \sum_j \phi_j b_{ji} [(\dot{h}_j - \dot{h}_i) + k(h_j - h_i)] \qquad (4)$$

Included is a spring constant, k, that scales the relative strength between the differences $\dot{h}_j - \dot{h}_i$ and $h_j - h_i$. The Filter Bubble Drag is a simple drag quantified as $-C_1 \dot{h}_i$.

The Bimodal Potential Well models the difficulty for a person to be between a *hipster* and *mainstream*. The potential function of hipster index, $U(h_i)$ is piece-wise infinite when $h_i < 0$ or $h_i > 1$, so Hipster values are bounded on $[0, 1]$.

$$U(h_i) = C_2 e^{-60(h_i - 0.5)^2}, \quad h_i \in [0, 1] \qquad (5)$$

The force on the hipster index for potentials are defined by

$$F_U(h_i) = -\frac{dU}{dh}(h_i) \qquad (6)$$

Equation (7) is derived from equation (5).

$$-\frac{dU}{dh}(h_i) \propto C_2(h_i - 0.5)e^{-60(h_i - 0.5)^2}, \quad h_i \in [0, 1] \qquad (7)$$

The final differential equation is the linear sum of the terms.

$$\ddot{h}_i = C_0 \rho_i \sum_j \left[\phi_j b_{ji} [\underbrace{(\dot{h}_j - \dot{h}_i)}_{\substack{\text{rate of} \\ \text{trend change}}} + k \underbrace{(h_j - h_i)}_{\text{perceived trend}}]] - \underbrace{C_1 \dot{h}_i}_{\substack{\text{filter} \\ \text{bubble} \\ \text{drag}}} - \underbrace{C_2 \frac{dU}{dh}}_{\substack{\text{bimodal} \\ \text{force}}} \right.$$
$$(8)$$

IV. RESULTS

Given the largely random nature of our model, results vary widely in depictions of social dynamics. It's important to note that our model treats hipsters and mainstreams symmetrically about the $h = 0.5$ line. For every result there exists an exact counterpart, $(h_i, \dot{h}_i) \to (1 - h_i, -\dot{h}_i)$. The parameters of ϕ_i and ρ_i are initialized from a uniform random distribution between 0 and 1. $h_i(0)$ was uniformly distributed between 0 and 1. $\dot{h}_i(0)$ was uniformly distributed between $-\frac{1}{16}$ and $\frac{1}{16}$. C_0, C_1, & C_2 were varied between 0 and 6. And k between 0 and 2. Simulations of the model with the population size ranging from 5 to 100 were performed.

Behavior remained relatively consistent between simulations with different parameters. The majority of simulations showed individuals converging smoothly toward being hipster or mainstream. All solutions with non-zero populations in both subgroups showed unstable behavior.
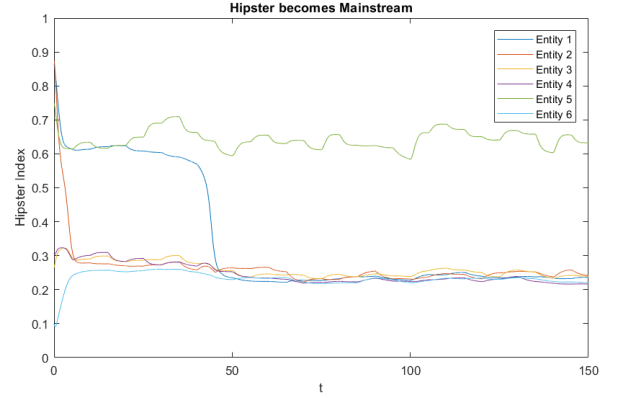

Fig. 2: After several randomization periods, a hipster is converted to being a mainstream due to social influence.

V. CONCLUSIONS

We use graphs, consisting of nodes and edges, to represent the social structure (people and relationships between them). Three different indices (Hipster index, Influencing index, and Responsiveness index) are used to model the inherent characteristics of a person. And two directional, weighted edges are used to model two way relationships between individuals. To simulate the interactions between people, two types of Randomness matrices are introduced. They account for the social dynamics such as exchange of information, change of bonds between people, and also the change in a person's characteristics caused by those interactions.

How a person behaves in the group depends on 3 factors: how the general trend changes, how much the change impacts the person (determined by bond strengths and influence index), and in what way the person responds to the impact experienced (determined by responsiveness). The system of differential equations relates these factors to the Hipster index. The solution of the system shows the change in the appearance of the population over time.

Based on the behavior we observe on finite time intervals, most solutions converge to one subgroup quickly. So everyone looks exactly the same in a very short span of time. However, in the majority of other simulations, the individuals reach a quasi-stable state where more than one subgroup exists over a substantial time frame. It seems likely that after infinite time, the individuals would converge to one subgroup due to random effects. This part of the time, the individuals in the subgroups will look quite similar and will likely show no indication of changing drastically. One parameter that has a large effect on solution behavior is C_2, the coefficient of the bimodal potential well. At low C_2, convergence is inevitable. At high C_2, convergence becomes increasingly unlikely because of the forces pushing individuals away from crossing $h = 0.5$.

REFERENCES

[1] Dunbar, R. I. M. (1992). "Neocortex size as a constraint on group size in primates". Journal of Human Evolution. 22 (6): 469–493. doi:10.1016/0047-2484(92)90081-J.
[2] "Are we stuck in filter bubbles? Here are five potential paths out". Nieman Lab.
[3] https://phys.libretexts.org/Courses/University of California Davis/Modern Physics/One-Dimensional Potentials Infinite Square Well.