PROBLEM C

Executive Summary: Movement of An Object in Microgravity Environments

In this problem the ultimate goal is to find the smallest size of asteroid that we can jump around on. There are two ways that we came up with for optimizing the asteroid's size. The first is the fragility of the probe. Violent landing/launching would damage it. The other is based on escape velocity. Any obstacle that would require a jump where the exit velocity equals or exceeds the escape velocity of the asteroid is problematic.

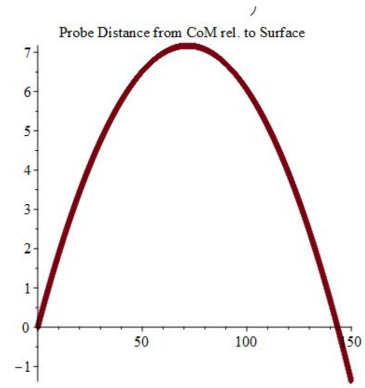
We made several assumptions in order to start this problem. Firstly, the problem tells us that the probe is fragile, so we need to worry about the acceleration it undergoes. Here, we looked up the amount of Gs the Mars Rover could handle (9Gs) and thought 4Gs would be reasonable for the asteroid probe. Secondly, we made an assumption about the asteroid. Since we are landing on it, we are going to assume that it is not spinning very fast. At this short of a radius and speed, we are going to ignore the Coriolis Effect. The probe should act approximately the same at any latitude. When moving arbitrarily through an asteroid field, the force on the probe is the sum of all the gravitational forces plus any thrust from the probe (in this problem, it is zero). Finally, we also assume that since this is the asteroid belt, other objects are far enough away to have negligible gravitational pull, so that the probe is only affected by the asteroid on which it lands.

The first constraint we found is on the size of the asteroid. Using the 4Gs acceleration (40 m/s^2) as an upper limit, kinematics equations, and potential energy from gravity, we found that to keep the max impact velocity below 8.9 m/s and above 1 m/s, the radius of the asteroid needs to be between 1.6 and 2.6*10^6 Km. We used the escape velocity equations and assumed for the first part that the probe decelerates at a constant 40 m/s^2 rate over 1 meter as it impacts the surface. If there is an acceleration greater than this cause by a greater impact velocity, then the probe will break and not be usable any more. The 1 m/s limit was created because we want the probe to be able to jump up at least at a rate of 1 m/s without having to worry about leaving the asteroid entirely. While the math for this assumed a spherical asteroid, it is approximately the same for an arbitrarily shaped asteroid as the force of gravity only acts from the center of mass.

For the equation of motion for jumping, it was more involved. Initially, we wanted to use the coriolis equations to account for potential rotational motion, but with the given time constraints we were unable to include this aspect. Therefore, when in the air, the only force acting on the asteroid is the force of gravity, represented by

$$Fg := z \rightarrow \frac{G\, m\, M}{(r+z)^2}$$

Where r+z is the distance from the probe to the center of mass and r is the distance from the center of mass to the surface. R is not necessarily a constant value, it is dependent on the shape of the asteroid. Therefore, to use just Fg in the equation of motion, we use z as a position, and then the x and y coordinates change relative to the surface. More specifically, x,y,z are orthogonal but r is not always orthogonal to x,y, or z. This result is not completely intuitive because as we live on earth we are accustomed to being pulled straight to the ground at all times. However, on a rectangular asteroid, gravity will act at an angle pulling you down and sideways towards the center of the asteroid. Motion with respect to time of the probe with some initial x velocity looks like the graph below:



This is semi-parabolic, but has some adjustments. This was found using maple to solve the ODE numerically. I tried several different methods of solving it analytically, but did not get any useful results. This numeric plot is as expected, and as the initial velocity increases the height and time of flight increases, until it is greater than the escape velocity, when the object would leave the orbit entirely.

Our probe has 4 springs, all oriented at 45 degrees from normal, and they are all 90 degrees from each other. The angles are constant but the springs can be loaded different amounts, allowing an arbitrary velocity in any direction as long as the radial component is less than the escape velocity. The energy from the springs is transferred instantaneously into the velocity of the probe, so the equation of motion is dependent on the force of gravity.

The escape velocity is defined as the initial velocity of the probe required for the probe to leave the asteroid's gravitational well. If the velocity of the probe exceeds this, it will leave the asteroid, which we do not want. Therefore, the size of an obstacle the probe can jump is a function of the size of the asteroid and some part of the aspect ratio.

If the asteroid fits within the radial constraints, then the probe can jump arbitrarily high in any direction safely. Assuming that the suspension of the probe can absorb the entire impact and does so over 1 meter. These could be hydraulic or spring based. If the asteroid is larger than the radial constraints give, then it can jump in any direction as long as the final velocity is less than 8.9 m/s.

Factors that limit the size of the asteroid we can land and jump on are the size of the asteroid and the durability of the probe. If the probe is reasonably durable, it can land asteroids as small as 1600 meters and as large as 2.6*10^9m. Effects of the shape of the asteroid would be a changed distance from the centroid, which would mean a jump is more stable in lower regions of the asteroid.

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To account for rolling the probe (we originally had an idea for this, which our presentation will reflect) we will have a mechanism where the spring will move a gear, which moves a horizontal spring, transferring energy to a horizontal spring. This last spring will spin a wheel. Sorry for the ambiguity.