Executive Summary

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Problem B: Movement of An Object in Microgravity Environments

Landing Model:

Specifications:

- Minimum dimensions of the target asteroid
- Land the probe as gently as possible so as not to damage it. For our model this means minimal bouncing.
- Do not want velocity of the bounce to exceed escape velocity
- Minimize bouncing, tumbling, and skidding to land as close to target as possible

Simplifying Assumptions:

- Relatively flat landing surface
- Angle (Ø) of launch is approximately 90° relative to the surface of the asteroid
- Mass of the probe is approximately 1.1 kg
- Air resistance is negligible
- Height of launch from mothership is 60 m
- Angle (θ) of launch is 0

Variables and Parameters:

Parameter	Variable		Units
Gravitational constant	G	6.67E-11	Nm²/m⁻²
mass of asteroid	m_a	2.05E+10	kg
mass of probe	m_p	1.10E+00	kg
radius of asteroid	r	150	meters
gravity of asteroid	g	6.07E-05	m/s^2
coefficient of restitution	μ	0.9	
Δt	step	0.01	s

Initial Conditions		Units
Z	60	m
X	0	m
Y	0	m
Vx	0	m/s
Vy	0	m/s
Vz	0	m/s
θ	0	degrees
ϕ	0	degrees

Equations: $x,y,z = V_{x,y,z}t - gt^2$

Position:

$$\frac{dx}{dt} = V_x = V\cos\theta\sin\emptyset$$
$$\frac{dy}{dt} = V_y = V\sin\theta\sin\emptyset$$
$$\frac{dz}{dt} = V_z = V\cos\emptyset - gt$$

Velocity:

$$\frac{dV_x}{dt} = 0$$
$$\frac{dV_y}{dt} = 0$$
$$\frac{dV_z}{dt} = -g$$

Analysis: Simple kinematics can be used to describe the probe as a particle in freefall from the deployment of the probe from the mothership while minimizing bouncing of the probe. The gravity of the asteroid will determine the magnitude of tumbling after the probe contacts the asteroid. Spherical coordinates were used to determine the position of the probe at any time, where the x,y plane is the surface of the asteroid.

Movement Model:

Specifications:

- Move the probe using a spring for propulsion (no thrust)
- The probe should be able to navigate uneven terrain
- Choose some theta for spring launch that does not exceed escape velocity
- Find limits of navigation for the probe

Simplifying Assumptions:

- Model spring propulsion through one jump at a time
- Adjust theta to be small relative to zero
- Assume friction from surface of the asteroid

- Spring can be compressed from 0 to 2 cm to influence magnitude of jump
- Assume mass of the probe is 1.1 kg
- Assume angle of impact is approximately the same as angle of launch

Variables and Parameters:

Parameters	Variable		Units
Gravitational constant	G	6.67E-11	Nm²/m⁻²
mass of asteroid	m_a	2.05E+10	kg
mass of probe	m_p	1.10E+00	kg
radius of asteroid	r	150	meters
gravity of asteroid	g	6.07E-05	m/s^2
spring compression	s	0.02	meters
angle of launch	θ	30	degrees
spring constant	k	50	N/m
coefficient of kinetic friction	µ	0.0001	
launch velocity	$V_{initial}$	1.35E-01	m/s
Δt	step	0.01	s

Initial Conditions		Unit
$V_{initial}$	1.35E-01	m/s
Z	0	m
X	0	m

Recursive functions using Euler's Method:

Launch:

$$x_{k+1} = x_k + (V_{xk})\Delta t$$
$$V_{x,k+1} = V_{x,k}$$
$$z_{k+1} = z_k + \left(V_{z,k} - g\Delta t'\right)\Delta t$$
$$V_{z,k+1} = V_{z,k} + (-g)\Delta t$$

Skidding:

$$x_{k+1} = x_k + \frac{(V\cos\theta)^2}{2\mu g}\Delta t$$
$$V_{x,k+1} = V_{x,k} + (-\mu g)\Delta t$$
$$z_{k+1} = z_k$$
$$V_{z,k+1} = 0$$

Analysis: Propelling the asteroid using a spring can also be modeled with kinematic equations. The maximum height and total distance traveled is determined by choosing values for an initial launch angle (θ), a spring constant, and spring compression. Varying the spring compression will alter the initial velocity and thereby alter the final position of the probe and distance traveled per hop. The probe used for modeling was assumed to be of the dimensions described in articles regarding OWL and HIBOU, the probes dropped on Ryugu[1] by Hyabusu2.[2]

It was found that an asteroid with a radius of approximately 150 meters and a mass of $2.045 \cdot 10^{10}$ kilograms would have enough gravitational force to land a probe weighing 1.1 kilograms. These mass and radius values are averages found in order to model the asteroid spherically. This was found using the movement model. The initial launch angle (θ) was set to 90 degrees. A density was kept within 10% of the average density found on type C asteroids[4], and the ratios were varied to find the smallest dimensions at which the probe's initial velocity at launch would not exceed the escape velocity of the asteroid. The probe may maneuver on the asteroid by a spring hopping mechanism. The probe may clear or land on obstacles of maximum height of 18.7 m and has a maximum range of 129.8 m. Other obstacles may be maneuvered by adjusting the compression of the spring with the max spring compression of 2 cm.

References

[1] Wall, Mike, "Japanese Spacecraft Successfully Snags Sample of Asteroid Ryugu," space.com, 22 February 2019, https://www.space.com/japanese-asteroid-probe-lands-ryugu.html. Accessed Nov 2019.

[2] Wall, Mike. "Meet OWL and HIBOU! Japan's Asteroid Hoppers Get New Names." Space.com, Space, 15 Dec. 2018, www.space.com/42753-japan-hayabusa2-asteroid-rovers-names.html.

[3] "Asteroids: In Depth." NASA, NASA, solarsystem.nasa.gov/asteroids-comets-and-meteors/asteroids/in-depth/. Accessed Nov 2019

[4] Yoshimitsu, Tetsuo et al. “Advanced robotic system of hopping rovers for small solar system bodies.” (2012). Accessed Nov 2019