SCUDEM Problem B Executive Summary

Jacob Scott, Youngin Seo, Jason Zheng

9 November 2019

New York University, Coach Mutiara Sondjaja

1 Safely Landing Probe on Asteroid

The goal of section one is to write a function that models safely landing a probe on an asteroid. We assume that the mass of the probe (m), its initial velocity (v_0), and the angle between a line pointing straight down at the asteroid and the line pointing straight at the landing point (θ) are known values. From integrating functions such as Newton's Second Law of Motion and other common physics equations, we produced the formula

$$y_0 = -\frac{1}{2}a_{thrust_y} sin(\theta)\left(\frac{4y_0^2}{v_{y_0}^2}\right) + \frac{1}{2}g\left(\frac{4y_0^2}{v_{y_0}^2}\right) + 2y_0$$

Solving for $a_{thrust} sin(\theta)$ gives

$$a_{thrust} sin(\theta) = \frac{v_{y_0}^2}{2y_0} + g$$

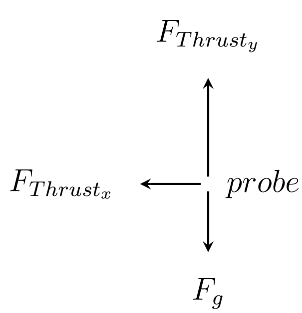
Similarly, a function for the needed acceleration in the x direction would be

$$a_{thrust} cos(\theta) = -\frac{v_{x_0}^2}{2x_0}$$

Multiplying through times the mass of the probe produces a function that gives the required force in the x and y direction for a safe landing.

$$F_{thrust_x} = -m\frac{v_{x_0}^2}{2x_0} \implies F_{thrust_y} = m\frac{v_{y_0}^2}{2y_0} + mg$$

FBD, "Probe Entering Landing Sequence"



2 Moving Probe to New Location

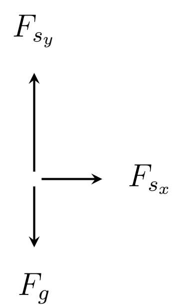
Goal: To create a function that models bouncing using springs. The probe will bounce multiple times before reaching the desired displacement x. The force exerted in each subsequent bounce will be a function of the force exerted in the previous bounce. To find the force needed to travel a distance x, we wrote a recursive relation relating each bounce by a constant μ. After finding a closed formula for the relation, we were able to integrate common physics equations and substitute in F_s. Given a desired x displacement, the function should produce the required direction and magnitude of the initial spring force vector.
Function that gives x displacement in terms of a beginning vector $F_s cos(\theta)$

$$x = \frac{(v - v_0)[(v - v_0) - \frac{F_\mu (v - v_0)}{F_s cos(\theta)} + 2v_0]}{2a} + x_0$$

Solving for F_s gives

$$F_s = \frac{-F_\mu (v - v_0)}{cos(\theta)\left(\frac{2ax}{(v - v_0)} - v - v_0\right)}$$

FBD "Probe Beginning First Bounce"



3 Finding Minimum Dimensions of Asteroid

Our goal is to define a formula that gives a minimum set of dimensions for the asteroid. Given the desired force of gravity between the asteroid and the probe, we used $Fg = G\frac{m_1 m_2}{d^2}$ to calculate the distance d. From this value d, we can find the radius r of the asteroid such that ($d - 10000$ meters $= r$). We understand that asteroids do not often have uniform shapes, so the "radius" should be the distance from the center of the asteroid to the landing point on its surface. Therefore, an appropriate formula would be

$$r = \sqrt{\frac{Gm_1 m_2}{F_g}} - 10000 \text{ meters}$$