Executive Summary

Problem B
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1 INTRODUCTION

We have been asked to provide guidance in helping find a new asteroid on which to land a probe. We provide this guidance by describing the dimensions of possible asteroids and the possible limits to what area can be explored.

2 ASSUMPTIONS

We begin by assuming our probe has a mass of 1 kg. We assume that the potential energy in the spring used for jumping is constant. We also assume that it is small enough so that the probe remains close enough to the surface to assume a constant acceleration due to gravity. Furthermore, we assume that the angle at which the probe jumps is at a constant angle of 45°. As for the deployment, we assume that the mothership of the probe is in a geosynchronous orbit with the asteroid; static with respect to the target landing region. We assume that the target landing region is level with respect to the asteroid's center of gravity. As for the asteroid, we assume that it is has a slow enough rotation so that the rotation has a negligible effect. We also assume that the asteroid has uniform density ρ.

3 SIZE OF THE ASTEROID

The probe's spring-based mode of transportation wholly relies on gravity to return it to the surface after each jump. As a consequence, the maximum allowed speed is the escape velocity from the surface. If the speed were allowed to exceed the escape velocity, it would not be able to return to the surface. Because of this fact and the assumptions we made, we can describe the size of the smallest asteroid in terms of the potential energy of the spring. In other words, for a given spring we can find the smallest asteroid in which the speed it produces does not exceed the escape velocity from the surface.
By assuming the jump angle is constant at 45°, we can describe the maximum speed produce by the spring in terms of the potential energy of the spring, u:

$$V_{vertical,max} = \sqrt{u} \tag{1}$$

Comparing equation (1) to the escape velocity produces the following,

$$V_{vertical,max} \leq V_{escape} = \sqrt{\frac{2GM}{R}}, \tag{2}$$

where G is gravitational constant, M is the mass of the asteroid, and R is the mean radius of the asteroid. Given a density ρ, we can write M in terms of R, and vice versa. In doing so, we can write the minimum radius R in terms of the potential energy of the spring:

$$R_{min} = \gamma\sqrt{u} \tag{3}$$

where $\gamma = \sqrt{\frac{3}{8G\rho\pi^{1/3}}}$. Equation (3) gives us the following DE,

$$\frac{dR}{du} = \frac{\gamma}{2\sqrt{u}} \tag{4}$$

4 DEPLOYMENT

The impact velocity is given by

$$V_{impact} = \sqrt{2GM\left(\frac{1}{R} + \frac{1}{R+d}\right)}, \tag{5}$$

where d is the altitude of the drop. Since $\lim_{d\to\infty} V_{impact} = V_{escape}$ which is much less than the velocity required to damage the probe, we can choose any altitude d to ensure that the probe experiences a gentle and accurate landing.

5 TRAVERSAL

Given a chosen spring and the assumption that the angle of the probe's jumps are at a constant angle of 45°, we can write the maximum horizontal and vertical distance of the jumps.

$$x_{max} = \frac{2\gamma^2 u^{3/2}}{\sqrt{2}GM} \tag{6}$$

$$y_{max} = \frac{\gamma^2 u^{3/2}}{\sqrt{2}GM} \tag{7}$$

Equation (7) determines the highest cliffs we can climb.

6 ADDITIONAL ISSUES

What role does the size and shape of the asteroid play in your model? The size of the asteroid is constrained by the spring in the probe. The shape of the asteroid determines where the probe would land (a target region allowing us to assume spherical surface gravity) and how to traverse the surface.

If the size of the asteroid changes what impact does that have on your predictions? A bigger asteroid would be easier to land on but would require more energy to move across. As the size of the asteroid increased your energy required to move larger distances would increase along with the gravitational pull of the asteroid. As the size of the asteroid increased bouncing would become less of a factor due to acceleration of gravity increasing.