United States Military Academy
Team 4
Kyle Spinelli, Maxence Sutton, Alexander Liesen
Coach CPT Ivan Bermudez
Problem B

Initial Model: Our model is based on safely landing and transporting a 1kg space probe on a small asteroid. This mass This is based on the *Huyabusa* mission in which the probe, *MINVERVA*, is about 1 kg (Yoshimitsu 2). To land the probe on the asteroid, we rely on a spring with a spring constant $k_{landing\ spring} = \frac{29{,}400\ N}{m}\ (spring\ constant\ of\ landing\ spring\ in \frac{N}{m})$, based on possible spring constants for a small metal wire spring, to absorb the impact of the initial impact ("Round Wire Springs"). This is modeled by the following second order, linear, homogenous ordinary differential equation:

$$m\frac{d^2x}{dt^2} + \sigma\frac{dx}{dt} + kx = 0$$

Where m is equal to the mass of the probe, x is equal to the distance that the spring being used to land the probe safely is compressed or stretched from equilibrium, and σ is a damping constant. In order to overdamp the system and bring the spring-probe system to a quick stop, we selected a damping constant of $\frac{150Ns}{m}$. We found a closed form solution to the differential equation, $x = e^{-64.4t} - e^{-10.6t}$,with real, non-repeated roots by assuming that k was at the lower end of our range of possible spring constants ("Round Wire Springs"). This safely lands our probe.

The probe will be moved using a spring to push off the surface of the asteroid, sending the probe airborne in order to move it. The spring will have a tire-like object with a high coefficient of friction attached at the end in order to launch the probe. The spring will be compressed using solar electrical energy generated by solar panels on the probe. The object attached to the spring will exert a force upon the surface of the asteroid at 45° in order to maintain a balance between the ability of the probe to push off from potentially rugged surfaces at an effective angle, and the ability of the probe to achieve forward motion. The spring will have a spring constant $(k_{movement\ spring}) = \frac{4{,}900\ N}{m}\ (spring\ constant\ of\ movement\ spring\ in\ N/m)$ based on possible spring constants for a small metal wire spring ("Round Wire Springs"). The spring will also be 4 cm long and compress a maximum of 3 cm due to the constraints of the size of the probe. We assumed the length of the spring based on the dimensional limitations of the Minerva spacecraft, which was 12 cm in diameter and 10 cm in height. A gyroscopic control will be used to orient and aim the spring, which will also be powered by the solar panels.

The size of an ideal asteroid could be determined by the relationship between the escape velocity v_e, or the velocity at which the probe would leave the influence of the asteroid's gravitational field:$v_e = \sqrt{\frac{2Gm_{asteroid}}{r}}$ where G is the gravitational constant, $m_{asteroid}$ is the mass of the asteroid, and r is the radius of such an asteroid. However, if we substitute an assumed value for the density of an asteroid, $\rho_{asteroid} = \frac{2{,}000\ kg}{m^3}\ (density\ of\ the\ asteroid\ in \frac{kg}{m^3})$, $\rho_{asteroid} * r^2 = \frac{m_{asteroid}}{r}$based on a commonly assumed density of an asteroid, we are able to use the geometric relationship between density and mass and volume to determine that the that the radius (r) of our ideal asteroid would be 3441 m and the ideal mass $(m_{asteroid})$ would be $6.825*10^{14}$ kg. We found r after assuming that we would have a spring constant k_{motion} of 29400N/m, a spring compression distance of .03m, and an 45° angle of launch perpendicular to equivalent gravitational lines. Using $\frac{1}{2}k_{motion}x^2 = \frac{1}{2}m_{probe}{v_e}^2$We can use the mass and radius of the asteroid to calculate a surface acceleration due to gravity a_g, which we can then assume is constant throughout the falling of the probe to the surface of the asteroid to simplify kinematic calculation. The falling of the probe can be modeled by the following differential equation:

$$\left(\frac{dx}{dt}\right)^2 = {v_0}^2 + 2\frac{d^2x}{dt^2} * h$$

Where h represents the distance that the probe has fallen at any given time. The probe will be released without any initial velocity, so $v_0 = 0$. By setting the final velocity at which the probe can strike the planet, $\frac{dx}{dt}$, equal to 2 m/s, or the maximum velocity at which a soft landing is made, we are able to determine that the maximum height from which our probe can be dropped by the carrier satellite is 520.2 meters above the surface of the asteroid ("Chandrayaan-2").

Our model is limited by several assumptions. The first of these is that the gravity is the same everywhere as it is calculated to be at the surface. Another of these is that the density of the asteroid is uniformly 2000kg/m^3, while in reality the mass and density of the asteroid would likely not be this simple for the asteroid.

The celestial body Diemos confirms our result. It has very similar mass, radius, and acceleration due to gravity at the surface to our optimized asteroid, which indicates that such objects exist, and it helped to provide a sense of validity to our numbers ("Ellipsoid"). Using the kinematic equation $d = \frac{v^2\sin(2\theta)}{g}$ we are able to determine that the probe will travel 1042 meters with each jump, making our rate of movement $\frac{1042m}{Charge\ Time\ of\ the\ Spring}$. The time it takes to charge is equal to the time in which capacitor compresses the piston and the piston charges the spring. Using the kinematic equation $H = \frac{v^2 sin^2(\theta)}{2a_g}$ where H is the height of the jump, we are able to determine that our probe could clear obstacles up to 260m in height while traveling.

References

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