SCUDEM CHALLENGE

Problem B

Alexandra Evans, Henry Brunnett, Joseph Patterson

Coach: David Kaplan

York College of Pennsylvania

Executive Summary

The goal of this project was to find the smallest sized asteroid that a probe could land on successfully using a spring while also keeping the number of jumps to a minimum. Another goal of this landing was to have the final resting place of the probe as close as possible to the designated landing area. The second part of the challenge is to design a method for the landed probe to move to another location with the use of a spring while minimizing the number of jumps.

To start out on this challenge, the team started by isolating some of the variables involved in the problem to begin with a simplified version. One of the challenging characteristics of the asteroid landing is the uneven surface. On a flat surface, this problem may not be as complicated, but the asteroid is assumed to be quite rugged with both ravines and steep cliffs the probe must be able to navigate. Initially assuming the surface is flat allowed the team to start small adding details and complexity as the project progressed.

The team first looked at basic velocity and spring equations. The velocity of the probe could be found using the equation $m\frac{dv}{dt} = mg - bv$ **.** In this equation the variables represent, m = mass, g = gravity constant, b = drag coefficient, and v = velocity. When solved for velocity the equation is $v = \frac{mg}{b} + Ce^{\frac{-b}{m}t}$ **.** Using this equation to find velocity will allow the team to determine the force on the asteroid from the probe's initial impact. With this value the team can then find the compression of the spring which is mounted on the probe. This distance can then be plugged into the spring mass equation $my'' + by' + ky = F_{ext}$ to find the resultant force and displacement of the probe. The ideals behind this can be used for each time the probe impacts the asteroid before final landing.

The scenario the team looked at was more complicated than an ideal situation, meaning there are several variables that must be looked at. First, the probe may not approach the surface at an ideal angle, which could result in it bouncing away from the predetermined location. Alternatively, the probe may approach correctly, but since the terrain is assumed to be quite rugged, it may strike the surface and bounce off at a different angle, causing it to land at a final location quite far from the intended spot. Both of these variables could cause damage to the probe as it could strike the asteroid with different parts that may not have the same durability.

At this point, the team stepped back and considered the initial problem again. Since the goal of the first part of the problem was to find the minimum size of the asteroid that the probe could land on, the team decided to find out what varied with asteroid size. The major component to consider here is gravity. The force of gravity varies based on the mass of an object and the distance between the two objects. As the asteroid size decreases, the force of gravity decreases. This has a negative and positive effect. The lower the gravity, the less of a chance of damage to the probe. Also a low gravity asteroid would decrease the amount of jumps required to land. The negative of the lower gravity is that the probe could bounce too high and leave the gravitational pull of the asteroid. Once the probe left the gravitational field the probe would be lost moving through space.

After some research, the team found this differential equation that determined the gravity of an asteroid:

Therefore, the asteroid gravity is obtained as follows:

$$U(r,\theta,\phi) = \frac{u_a}{r}\left\{1 + \Delta\left(\frac{R_0}{r}\right)^2 + \Gamma\left(\frac{R_0}{r}\right)^4 + O\left(r^{-5}\right)\right\},$$

where

$$\Delta = \left[\frac{1}{2}C_{20}\left(3\sin^2\phi - 1\right) + 3C_{22}\cos^2\phi\cos 2\theta\right],$$

$$\Gamma = \left[\frac{1}{8}C_{40}\left(35\sin^4\phi - 30\sin^2\phi + 3\right) + \frac{15}{2}C_{42}\cos^2\phi\left(7\sin^2\phi - 1\right)\cos 2\theta + 105C_{44}\cos^2\phi\cos 4\theta\right].$$

Since it's a microgravity situation on an asteroid, this will give an accurate value of gravity to use in the other equations.

The original spring equation didn't take the rugged surface of the asteroid into account, which led to finding the following equation. $m\frac{d^2y_1}{dt^2} + \lambda(\frac{dy_1}{dt} - \frac{dy_2}{dt}) + k(y_1 - y_2) = 0$

The use of this equation allowed the team to add a level of complexity to the method, being one step closer to the actual solution.

Although a conclusion has not been reached as far as the minimum size of an asteroid the probe could land on, these equations helped us make some progress in how to go about solving a complex real world problem. These differential equations, especially the one that determined the gravity on an asteroid, could prove very helpful in an actual scenario as asteroid exploration becomes a sought after venture.

References:

Liu, Keping, et al. “Finite-Time Spacecraft's Soft Landing on Asteroids Using PD and Nonsingular Terminal Sliding Mode Control.” *Mathematical Problems in Engineering*, Hindawi, 22 Jan. 2015, www.hindawi.com/journals/mpe/2015/510618/#B20.

“Differential Equation - Modeling - Spring and Mass.” *ShareTechnote*, www.sharetechnote.com/html/DE_Modeling_Example_SpringMass.html.