Problem B:

Movement of Objects In Microgravity Environments

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Introduction:

The challenge consists of utilizing a spring to land a probe on an asteroid. It proposes mathematically modeling the equations to land in a designated zone with minimal amounts of bounces. The problem also asked for a way to model viable asteroids to land on due to the fact that asteroid surfaces vary greatly from one to another in elevation extremes, and roughness of surface.

The Approach:

The design of the model can be broken down to specific steps or stages. The pre-stage process seeks to model the viability of an asteroid's surface to be landed on by the probe. If the asteroid is determined to be viable to land; then a three-stage model of: the descent to the surface, impact and the projectile motion thereafter in repetition. The descent stage models the probe as it enters the gravitational field of the asteroid and the initial energies it comes with. The impact stage models the translation of those energies to the elastic potential of the spring. The launch stage is a projectile motion problem and models the probes forces for the next bounce oscillating between impact and launch finally models the probe's landing in its final, predetermined landing location.

Viability of an Asteroid:

Size, shape, and topology of asteroids vary widely from asteroid to asteroid. Not all asteroid surfaces would be viable to land a probe successfully on. So before approaching the problem of landing the probe the question of 'which asteroids are compatible' must first be answered. As stated asteroids vary greatly in size, shape, and topology, the main concern being topology. From a topological map many things can be determined such as the where peaks and valleys lie, the elevation of those peaks and valleys, as well as the grade of the slope of peaks to name a few. To determine the viability of the asteroid to land a probe on its surface the grade was studied the most. The idea was that too steep a grade or angle above (or below) the horizontal would create terrain that would be impossible to land the craft on or at the very least be very difficult to model accurately. With this idea the hypothetical classification system for the viability of an asteroid could be considered as the average grade of the asteroids surface. That is to say that, if an asteroid's average grade was too large that the majority of the surface would not be optimal to land on therefore, the asteroid would not be viable for the problem. To find the average grade or average steepness of a given asteroid the derivative of the grade (θ) with respect to the distance from the lowest elevation to the highest elevation. The differential equation is represented as $\frac{d\theta}{dD}$ where $\theta = \tan^{-1}(\frac{y}{\Delta x})$ and $D = \Delta x = x_f - x_0$ where Δx is the horizontal distance from the lowest elevation to the highest elevation.

To get the average steepness of the entire surface a grid needs to be applied to the topological map of the asteroid. Then, per grid square, the derivative of the grade, from lowest elevation to highest elevation, within the grid square, with respect to the change in distance of the two elevations ($\frac{d\theta}{dD}$) will give the change in grade of that grid square. Then to find the average divide the summation of all grid square changes and divide by the number of grid squares where the derivative of the grade is measured. This should give a relative average of the steepness of an asteroid. This average can then be equated to the viability of an asteroid to have the probe land on its surface.

The landing process:

Given a viable asteroid is located, and a landing zone is predetermined. The landing process must be carefully calculated. The initial energies and the nature of the spring play and important roles in the overall behavior of the motion in which the probe coupled with the spring will undergo (assuming that the change in overall height of the surface from the prescribed height is very minimal or that the surface in general is flat). Each stage is intricately intertwined in a dance of kinetic and potential energy exchanges. This dance is a ballet of transferring forces and vectors as to keep the probe locked into the gravity of the asteroid and land safely; considering the loss of energy in each stage.

Every journey starts with one step. Differential equations is no different, the initial value of an equation is the key to solving the unsolvable. Our journey to the asteroid surface starts with the most crucial step in the process. The initial velocity, and angle of descent is the heart of the equations. This determines how much force is applied to the spring mechanism, which in turn sets the initial launch velocity of the first bounce. This bounce is the second most crucial element of the stage. If the spring accelerates the probe at an initial acceleration greater than that of the gravity, the probe will be lost in space forever.

The probe will be travelling at a velocity relative to the asteroid. This kinetic energy is calculated as:

$1/2KE = 1/2mv\text{^}2$

Where m, is equal to the mass of the probe. The probe then enters the asteroid with an initial velocity, V, gravitational acceleration of the asteroid and begins to accelerate with respect to time. Acceleration is the change in velocity with respect to time. Integrating a(t) we find:

$\int mg\,dt = mgt + c = V(t)$ our equation becomes $V(t) = gt + Vinitial$

The velocity at impact, is equal to V(0). This gives us a start for our differential equation. Thus, if our initial velocity is very large our force of impact will be very large.

The kinetic energy transferred to the spring is directly related to the impact force. The kinetic energy is equal to the opposite of the spring potential energy shown.is the compression distance of the spring and k is the springs natural constant.

This acceleration when the spring expands will propel the probe at a velocity, $V_{launch1}$, the forces acting upon the probe is now gravity. shown in the equation above for our initial descent. To find the height and distance with respect to time we integrate v(t) getting:

$$\int mgt + v\,dt = -\left(\frac{1}{2}\right)gt^2 + vt + Yinitial$$

Solving for the derivative V(t) = 0 this will give us our new max height to apply our velocity equation. These two stages of impact and launch can be minimized due to specifics of the asteroid. The Energy Approach:

This approach uses the concept of the conservation and transformation of energy to innately describe the interaction and behavior of the probe in motion with the energy loss due to damping overtime and its distance traveled due to n amount of bounces. In this approach, the assumption that the surface it bounces upon was relatively flat helped in making the modeling of the probes motion easier.

At the initial stage, the probe is assumed to be at a height above the ground. This height was characterized as being the distance from the top of the probe just before falling to the bottom of the probe (note: not the bottom of the spring) before compression. This height enables us to find the potential energy and we summed that with the initial kinetic energy which depends on the initial velocity that the probe moved with. This initial velocity and height help define the initial amplitude of the motion after impact and the amount of bounces it takes to reach a value of ‘y’ which we consider to be the rest point of the probe. After the probe is let-off the initial energies get converted in a final kinetic energy which we use to determine the velocity before impact. $g = \frac{G*M_{asteroid}}{r^2}$

$(mgh + \frac{1}{2}mu^2 = \frac{1}{2}mv^2)t^{0-}$; $(\frac{1}{2}mv^2 = \frac{1}{2}kc_0{}^2 + energyloss)t^{0+}$; $\frac{dy}{dt} = \sqrt{u^2 + 2 * y'' * D_h}$

With this velocity before impact, we calculate the compression of the spring through: $c_0 = v_0\sqrt{\frac{m_\phi}{k}}$

And the amplitude is expressed as: $A_0 = \sqrt{y_0{}^2 + (\frac{y_0{}'}{\omega_0})^2}$; $\omega_0 = \sqrt{\frac{k}{m}}$

We portrayed the scenario of the probes motion to be that of an under-damped oscillation and to find the vertical position over time and to find the horizontal position, we said: $v_x = v_y \cot\theta; x = n * \frac{v_x}{t} + \frac{n}{2}$ and multiplied it by the n amount of bounce of the probe. We arrived at this relation by using the mass-spring oscillator equation and solving the characteristic equation for the underdamped case to get y(t). The function of y(t) is described as: $y(t) = Ae^{-\frac{\delta}{2m}*t}(\cos(\omega' t - \emptyset))$; $\omega' = \omega_0\sqrt{1 - (\frac{\delta}{2k})^2}$; $\delta =$

$\ln(\frac{y_{(t)}{}'}{y'(t+T_d)})$. => this id the logarithmic decrement over time and $n = \frac{(\frac{2t_n\omega'}{\pi} - 1)}{2}$

Energy Dissipated in Each Cycle: $T = \frac{2\pi}{\omega_d}$; $\zeta = \frac{\delta}{2k}$; $E(t) = \frac{1}{2}k * c_0{}^2 * e^{\frac{-4\pi\zeta}{\sqrt{1-\zeta^2}}}$

Energy Dispelled Over n Cycles: $DE_n = \frac{1}{2} * c_{n-1}{}^2 * e^{\frac{-4n\pi\zeta}{\sqrt{1-\zeta^2}}} * (1 - e^{\frac{-4\pi\zeta}{\sqrt{1-\zeta^2}}})$

Energy Dissipated in Successive Cycles: $\frac{DE_n}{DE_{n+1}} = e^{\frac{4\pi\zeta}{\sqrt{1-\zeta^2}}}$

It is noticed that without energy loss, the probe will continue to oscillate even though its amplitude inherently approaches Zero. In practical terms, this will not occur as energy is lost through the impact of the probe hitting the surface and is lost in the form of heat and sound or through air resistance. This loss helps the probe achieve a Zero rest position with respect to the vertical axis.