ES-C-New York University-Team-3

Coach Name: Lindsey Van Wagenen

Background Information: In this problem, we examine the unique reproduction procedure in a certain species of butterfly, known as *Pieris brassicae*. It is often challenging for butterflies of this species to find partners for mating. A common approach undertaken by the female butterfly to release chemical signals, which can be sensed by their male counterparts. When a male first senses this chemical signal, it reaches through to this female butterfly that released the signal, and mates with it. Anti-aphrodisiacs (referred to as the anti-signal in the entirety of this summary) are passed by males to females during mating to prevent females from remating, and to ensure female monogamy (Huigens et al). Unfortunately, for these butterflies, the anti-signals can also be sensed by parasitic wasps. These tiny parasitic wasps can then ride on a mated butterfly to the host plant, and parasitise her freshly laid eggs (Huigens et al). These two effects lead to competing pressure on the butterfly population. For the male butterflies, it makes it more likely for them to fertilise the eggs, and the females can focus on the most advantageous position to lay the eggs on, without being bothered by other males. On the other hand, the anti-signals make it more likely for the butterfly eggs to be attacked by the parasite eggs, and therefore the male butterflies are under strong selective pressure to minimise the use of the anti-signal.

Problem Statement: Develop a mathematical model to determine the interaction within *P. brassicae* (a kind of butterfly) species as well as between *P. brassicae* and the parasitic wasps. What is the best balance for this system and what is likely to happen in the long run?

Redefinition of the problem statement: Based on the given question, questions arise as to what is meant by the "interaction" between two species. And what is meant by the best balance of the system? Furthermore, how are these quantities mathematically quantified to make sense as in the case of our model? Firstly, we define what these terms mean in our model, and how we hope to achieve the goal of determining the best balance of this system. The interaction between male and female butterflies is quantified by a constant(l), which gives us the number of male butterflies, interacting with the number of female butterflies. The interaction between the female butterflies and the parasitic wasps is given by the constant(k) which gives us the number of pregnant butterflies that a wasp attacks. Our model incorporates these constants into the equation, in order to help us determine the rate of change of the two populations over time. We simulate various different conditions, with varying values of l and k in order to determine when and at what interaction rate between the two species is there an equilibrium in the community. To that end, we make a few **assumption**s which are stated below as follows:

(1) There are only two species namely in the environment:the butterfly and the parasitic wasps.

(2) The only food source available to the parasite larvae, are the host eggs of the butterfly. Therefore, if all the butterflies die, the wasps will also perish.

(3) The butterfly has abundant food. In other ways, while the wasps are dependent on the butterflies as their primary source of food, this relationship is not two way, and the butterfly is not dependent on the wasps as a food source.

In our model, we initially start out with a simple prototype of the Lotka-Volterra equation, but modify it in order to model the complex multiplicity of interactions taking place in between the two species. The Lotka-Volterra equations give us relationship between the predator and pray, and is shown below:

$$\frac{dx}{dt} = \alpha x - \beta xy,$$
$$\frac{dy}{dt} = \delta xy - \gamma y,$$

where

x is the number of prey (for example, rabbits);

y is the number of some predator (for example, foxes);

$\frac{dy}{dt}$ and $\frac{dx}{dt}$ represent the instantaneous growth rates of the two populations;

t represents time;

$\alpha, \beta, \gamma, \delta$ are positive real parameters describing the interaction of the two species.

We build the relationship between the butterfly species and parasitic wasps by modifying this equation. We use W(t) to represent the population of parasitic wasps (1) and B(t) to represent the population of the host butterfly (2).

rate of change eggs deposited deaths of waspS

$$\frac{d\square(\square)}{d\square} = \square\square\square(\square)\square(\square) - \square\square(\square) \quad (1)$$

$$\frac{d\square(\square)}{d\square} = \square\square\square\square(\square)(1 - \square)\square(\square) - \square\square\square(\square)\square(\square) - \square\square(\square) \quad (2)$$

rate of change eggs deposited eggs being eaten deaths of butterflieS

In this model,

a= number of wasps attracted per female butterfly

b= number of wasp eggs laid per wasp

c= number of butterfly eggs destroyed by wasp eggs

α= a*b*c. [This means that if we performed dimensional analysis on a, b and c, λ would be equal to the number of butterfly eggs eaten by a wasp egg]

λ= a*b [If we do dimensional analysis on a and b, this would mean that **α** is cost(“overhead”) of producing wasp eggs per female butterfly]

P = 0.6 is the ratio of female butterfly to the total number of butterflies. Therefore, this constant has no units.

μ is the average number of eggs laid per female butterfly.

k is the interaction rate between the pregnant butterflies and parasitic wasps. It is important to be noted that here k, stands only for the number of pregnant butterflies, and not the whole butterfly population. As was noted above and in the paper by Huigen et al, the parasitic wasps rides only on non-virgin female butterflies. And hence the only interaction that is of concern to us in this paper, is that between the wasps and the female butterfly.

l is the interaction rate between male and female butterfly, and we keep it variable also to see how the rate makes influence.

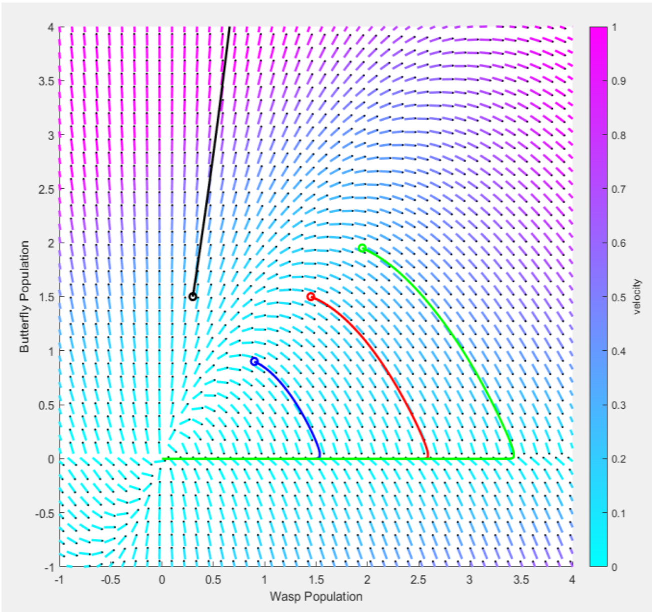
γ = 0.2 is the death rate of butterflies.

β = 0.4 is the death rate of wasps.

k and l are two variables. They are kept as variables so that we can determine how the different interactions affect the long run behaviour of the entire model. The two key variables **k** and □ are related because there is a trade-off with respect to the “anti-signal”, released by male butterflies to successfully fertilize the eggs but at the same time it attracts wasps . Therefore, if **k** goes high, □ must increase automatically, because if the interaction rate between male and female butterflies is high, more anti-signal is released, as a result of which more wasps are attracted to pregnant female butterflies. Furthermore, if k is low, this means that there is less interaction between males and females, which means that if fewer males and females mate, the only food source for our wasps(one of our assumptions) decreases, and hence eventually the wasps will perish. This is also the case when we start out with a lower population of butterflies than wasps, or if the wasps consume way too many butterflies. In both cases, the wasps will eventually perish because there is no other way for wasps to get food.

Application:

We use MATLAB to model the phase plane of this differential equation system and keep all the constants set, modifying the value of **k** (interaction rate in two species) and □ (interaction rate within butterfly). Here is one sample with k = 0.4, l **=** 0.4 with four different initial cases.



Here are the different values that were chosen for the constants on an average case basis:

λ = 30

α = 60

P = 0.6

μ = 200

γ = 0.2 is the death rate of butterflies.

β = 0.4 is the death rate of wasps.

The plot can be explained because if we start with far more butterflies than wasps, and then even if a large amount of eggs are eaten by wasps’ larvae, since the population base of butterfly is large, the population of two species would grow up together. But if the starting amount of butterflies and wasps is similar, due to our settings of strong-egg-consumption of wasps’ larvae, the butterfly population goes down sharply whereas the wasp population experiences a slow growth and then drops as well because there is no food sources for wasp larvae (since butterfly dies out). Therefore, this is how we determine the interaction rate between the butterflies and wasp population, as stated in our objective as the “interaction rate”, and the long run behaviour of the environment has been predicted by generating multiple MatLab plots.

Conclusion and Sensitivity Analysis:

As we test more situations of k and □**,** we find that based on the constants we set up, no matter what value of interaction rate we put, there are only two possible endings, both dying out or both boosting up. Furthermore the model is obviously most sensitive to k and l, since these were our only independent variables in our equations.The approved constants were chosen purely by simulating different scenarios on Matlab, which for the constraints in the page limit, have not been included here.

But if we make the interaction rate □ within the butterfly species dynamic, the outcome was amazing, the two species end up in a dynamic balance. The details of the same have not been included in the summary, for lack of space.

Possible Improvements:

- **Problem 1**: Under certain initial conditions, butterfly and wasp population grows exponentially
- **Improvement 1**: Incorporate carrying capacity to the equation to model a more realistic scenario
- **Problem 2**: The model assumes that wasps only lay eggs when there is anti-aphrodisiacs, and that the only food wasps get are from the butterflies which in reality is not the case.
- **Improvement 2**: Add another term in the W(t) equation to have the wasp population's growth rate to be independent of the butterfly population.

References:

[1] "Chemical espionage on species-specific butterfly anti-aphrodisiacs by hitchhiking Trichogramma wasps, " Martinus E. Huigens, Jozef B. Woelke, Foteini G. Pashalidou, T. Bukovinszky, Hans M. Smid, and Nina E. Fatouros. Behavioral Ecology. Volume 21, Issue 3, May- June 2010, Pages 470–478, 11 February 2010. https://doi.org/10.1093/beheco/arq007 .