Problem C
Beth Jennings, Emily Branson, Bethany Witemeyer
Coach John Ehrke
Abilene Christian University

Modeling the Interaction Between Butterflies and Parasitic Wasps

The female butterfly of the *Pieris brassicae* species uses an aphrodisiac pheromone to attract males, which in turn use an anti-aphrodisiac to deter other males. This anti-aphrodisiac is beneficial to the butterfly population in that the female is more successful in laying eggs safely when she is not distracted by other males. However, this anti-aphrodisiac can be detected by parasitic wasps, which then hitchhike on the butterfly and lay their eggs in the butterfly's eggs. In this way, the butterfly population is negatively affected by the anti-aphrodisiac.

There are two species of wasp to be considered, one of which innately knows to hitchhike on butterflies which have the anti-aphrodisiac, and one species which is only drawn to the anti-aphrodisiac after one successful hitchhike ride on a butterfly with the anti-aphrodisiac.

There are several assumptions we made in the development of this model. First, we assumed that the butterflies would have abundant sources of food and would experience exponential growth in the absence of wasps. Second, we assumed that the butterflies were the only source of food for the wasps and that both species of wasps would die in the absence of butterflies. We also assumed that the butterflies could be modeled as a single species rather than modeling the male and female butterflies separately. Finally, we initially assumed that the two wasp species would not interact, but this assumption was later removed as we started looking at the model we developed. We used $P(t)$ to denote the butterfly population at a given time, t, $W1(t)$ and $W2(t)$ represented the *T. Brassicae* and *T. Evanescens* populations, respectively. Each population was given three parameters, one for the growth of the population in the absence of the other two populations and two for the interaction terms between each species and the other two. All of these parameters, with the exception of the growth parameters for the wasp populations, are functions of the amount of anti-aphrodisiac the male butterflies use.

The *Pieris brassicae* population is increased by a growth parameter which is reliant on the amount of anti-aphrodisiac used. The population is decreased by interactions between *Pieris brassicae* and both wasp species, and both interactions are modified by a parameter which is also modified by the amount of anti-aphrodisiac used.

The wasp populations increase by a growth parameter which is reliant on the amount of anti-aphrodisiac used, and also increases for every interaction with the butterfly population, again modified by a parameter reliant on the anti-aphrodisiac. The wasp populations decrease for every interaction between each other, because the wasp populations are in competition.

$$\frac{dP}{dt} = \alpha P - \beta_1 PW_1 - \beta_2 PW_2$$

$$\frac{dW_1}{dt} = -\varphi W_1 + \delta PW_1 - AW_1W_2$$

$$\frac{dW_2}{dt} = -\gamma W_2 + \lambda PW_1 - BW_1W_2$$

We solved the equations for equilibrium solutions. In this way, we can analyze the relationships between the parameters and how they affect the situation. There are multiple points at which the system is in equilibrium when any one population — *P. Brassicae*, *T. Brassicae* or *T. Evanescens*— is extinct. Because these solutions do not allow for an analysis of the benefits and detriments of the anti-aphrodisiac, we will focus not on these, but on the fourth solution that we found:

$$P = \frac{AB\alpha + A\beta_1\gamma + B\beta_2\phi}{A\beta_1\lambda + BA\beta_2\delta},\ W_1 = \frac{A\alpha\lambda - \beta_2\delta\gamma + \beta_2\lambda\phi}{A\beta_1\lambda + B\beta_2\delta},\ W_2 = \frac{B\alpha\delta + \beta_1\delta\gamma - \beta_1\lambda\phi}{A\beta_1\lambda + B\beta_2\delta}$$

As the butterfly population is the central focus of our analysis, we shall primarily discuss the equilibrium solution for *P*. Shown above, the butterfly population is a function of all of our parameters: A (wasp-wasp interaction), B (wasp-wasp interaction), α (butterfly growth), β_1(*T. Brassicae* - butterfly interaction), ϕ (*T. Brassicae* population decay), β_2 (*T. Evanescens*-butterfly interaction), γ (wasp 2 population decay), δ (butterfly-*T. Brassicae* interaction), and λ (butterfly-*T. Evanescens* interaction*)*. It is easy to see why manipulating any of these parameters would have impacts on the long-term butterfly population. The term $AB\alpha$ informs that as the base growth rate of the butterfly populations (α) and the prevalence of wasp-wasp interactions (interference between the two wasp species' feeding patterns, or A and B) increase, so too does the long-term butterfly population.

Additionally, it is interesting to note that we observe an overall rough $\frac{\gamma}{\lambda}$ and $\frac{\phi}{\delta}$ dependence for *P*. This fits what we would expect to observe in the physical situation, as a higher death rate of a predator species generally benefits the long-term population of the prey species, while the saturation of the environment with organisms leads to an increased interaction between the two populations, having the opposite effect.

From our model, we can see that there are more parameters which decrease the butterfly population than there are parameters that increase the butterfly population. Because each of the parameters is modified by the amount of anti-aphrodisiac used, we can draw from our model that a high usage of anti-aphrodisiac will tend to negatively affect the butterfly population. However, we know from the problem statement that the presence of the anti-aphrodisiac does bring some benefit to the butterfly population, so this cannot be the whole story. We hypothesized that, in small amounts, the presence of the anti-aphrodisiac benefits the butterfly population, while in large concentrations it serves to benefit the wasps. We predict that this would be represented in our model via our parameters β1, β2, δ, and λ. We predict the following:

$$\alpha \propto \kappa^n$$
$$\delta, \lambda, \beta_1, \beta_2 \propto \kappa^m$$

Where κ is some value representing the prevalence of anti-aphrodisiacs and $n < m$.

The long term behavior of our model shows that an equilibrium between the butterflies and both wasp populations can be reached. This is supported by the real-world situation in that these butterflies and wasps coexist in their environment. Moreover, the given research paper, *Chemical espionage on species-specific butterfly anti-aphrodisiacs by hitchhiking Trichogramma wasps*, suggested that the use of the anti-aphrodisiac had an overall negative impact on the butterfly population. This is reflected in our model.

Additionally, we were asked to consider the impact of a female butterfly being able to detect a male's propensity to use the anti-aphrodisiac. In our model, this propensity is represented by κ. We can see that the growth rate α of the butterfly population is most positively benefited when κ is nonzero but less than 1. This implies that the female butterflies should choose mates with a lower propensity in order to most positively impact the butterfly population.