SCUDEM 2019 Problem C, Chemical Espionage
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Executive Summary

INTRODUCTION

For this assignment, the task was to find a mathematical model to determine the trade-offs and balance between the competing interests of male and female *Pieres Brassicae (P. Brassicae)* Butterflies, as well as two species of parasitic wasp, *Trichogramma Brassicae (T. Brassicae)* and *Trichogramma Evanescens (T. Evanescens)*. These two species of wasp prey on the eggs of female *P. Brassicae* butterflies. The Lotka-Volterra predator-prey model was chosen to illustrate a balanced ecological system, where the population of butterflies varies proportionally to the population of the wasps, forming a periodic and cyclical relationship between the two populations. This model was chosen based on the expectation that the population of wasps and butterflies would fluctuate based on the opposing population, yet still establish an equilibrium as time progressed. The relationship between the wasps and butterflies could also be modeled by using a competing species model, but we chose the predator-prey model because the butterfly eggs are prey for the parasitic wasp larva.

LOTKA – VOLTERRA MODEL

The assumptions applied to the Lotka-Volterra model are as follows:

- Butterfly population always has an abundant supply of food.
- Wasps only form of procreation is via parasitic consumption of butterfly eggs.
- The rate of change of each population is proportional to its size.
- Environment has no effect on either population.
- Wasps have limitless libido.

In relation to the assigned problem, the following assumptions have been added:

- Butterflies have a higher growth rate, but wasps have higher interference rate.
- Both initial populations have some real value greater than zero.
- Both wasp populations can be considered one population for purposes of this model.
- All male butterflies use the anti-aphrodisiac through the mating process.

This rate of change in each population can be expressed with the equations:

$$\text{Butterfly: } \frac{dx}{dt} = ax - \alpha xy$$

$$\text{Wasp: } \frac{dy}{dt} = -cy + \gamma xy$$

Where a and c are the growth rates of their respective population without predation. α and γ are the interference rates that the opposing population employs against the respective population. If

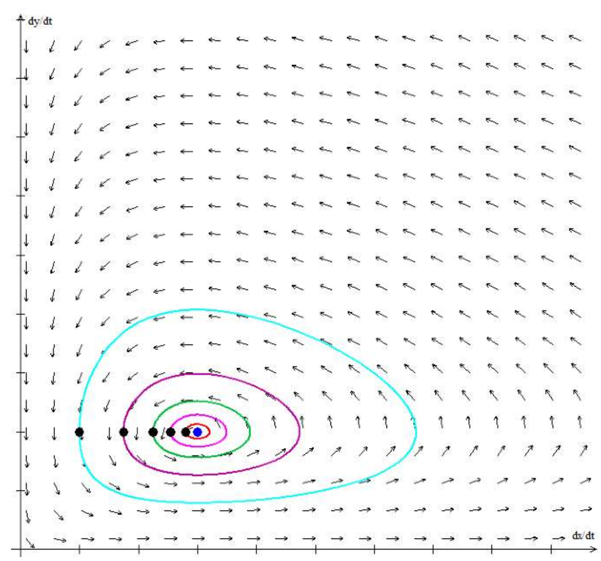

Figure 1: Cyclical Phase Portrait of Population Changes

each equation is set equal to zero, an x or y value can be found that corresponds to two unique equilibrium points. One equilibrium point, the origin, will be irrelevant since (0,0) would be the point of extinction for both populations. These equilibrium values can be calculated using:

$$x = \frac{c}{\gamma} \qquad\qquad y = \frac{a}{\alpha}$$

In *Figure 1* the rate of change in the population of wasps is compared to the rate of change for butterflies with respect to time. As stated, this is a cyclical phase portrait, meaning that both populations will continue to wax and wane as time progresses, so long as the assumptions mentioned previously hold true.

To find either population at any specific time, the equation below may be manipulated to obtain a solution. Notice that the two non-linear equations can be combined into a single equation of the form:

$$\frac{dy}{dx} = \frac{y(-c+\gamma x)}{x(a - \alpha y)}$$

This equation is now a separable ordinary differential equation. Solving this ODE, the general solution is:

$$a \ln y - \alpha y + c \ln x - \gamma x = K$$

Where K is some integration constant.

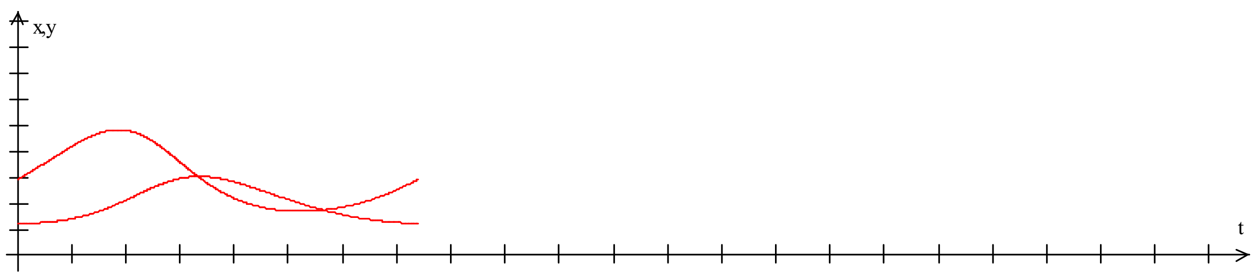

Figure 1: Butterfly and Wasp populations Over Time

In *Figure 2* each population can be seen individually with respect to time. The respective equations are:

$$y = \frac{a}{\alpha} + \frac{a}{\alpha}\sqrt{\frac{c}{a}} K \sin(\sqrt{ac}\, t + \phi) \qquad\qquad x = \frac{c}{\gamma} + \frac{c}{\gamma} K \cos(\sqrt{ac}\, t + \phi)$$

Where K and ϕ are determined by the initial conditions. Note that the wasp population will always be one-quarter periodic cycle out of phase from the butterfly population. This is due to the wasps' reliance on the butterfly eggs for wasp population growth.

Conclusion:

Based on the results found in the competing interests of the *P. Brassicae* butterflies and the parasitic wasp species *T. Brassicae* and *T. Evanescens*, it can be concluded that the dynamical population system will be in a stable and indefinite periodic cycle in the long run, with neither population decreasing to extinction so long as the stated assumptions hold true. It can also be stated that the cyclical period and carrying capacity of the two populations will be dependent on the initial values of the two populations. For this paper, the number of populations was reduced to two populations for simplicity, and ease of graphing in two-dimensions. However, the equation can be taken to degree *N* to account for other populations.