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Problem C: Chemical Espionage

For this model we are comparing the population of Butterflies (B) against the population of Wasps (W) as competing species described by the following system of equations:

$$\frac{dB}{dt} = B(\epsilon_1 - \sigma_1 B - \alpha_1 W)$$

$$\frac{dW}{dt} = W(\epsilon_2 - \sigma_2 W - \alpha_2 B)$$

In which the change in population of butterflies is determined by the population of butterflies and the population of wasps and vice versa. In this model, we can state that the butterflies and wasps are competing for the butterflies' eggs. The butterflies need the eggs for the species to survive and the wasps need the eggs as a food source for their own eggs to survive.

When solving for the equilibrium points of the solution we determine 4 points in total. Three of these points tend towards the extinction of either one or both species as shown:

$$(0, 0) \quad (0, K_W) \quad (K_B, 0)$$

In which K refers to the carrying capacity of each population $\left(\frac{\epsilon}{\sigma}\right)$. However, there is one equilibrium point in which both the butterflies and wasps can survive and coexist, and it is given by:

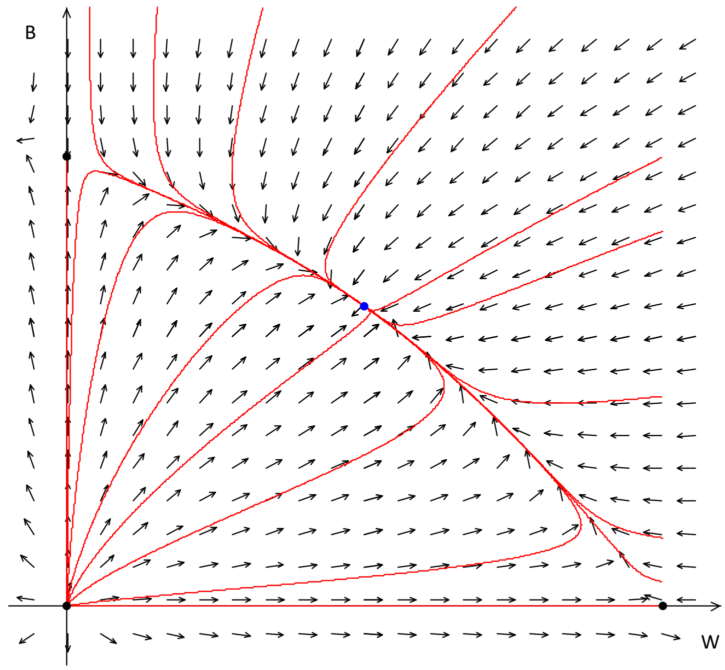
$$\left(\frac{\epsilon_2\alpha_1 - \epsilon_1\sigma_2}{\alpha_2\alpha_1 - \sigma_1\sigma_2}, \frac{\epsilon_1\alpha_2 - \epsilon_2\sigma_1}{\alpha_2\alpha_1 - \sigma_1\sigma_1}\right)$$

The best balance for this system occurs when both populations tend towards fourth point of equilibrium as time goes on. For this, we use a Jacobian Matrix of the system and solve for the eigenvalues given by the equation below.

$$\lambda_{1,2} = \frac{-(\sigma_1 B + \sigma_2 W) \pm \sqrt{(\sigma_1 B + \sigma_2 W)^2 - 4(\sigma_1\sigma_2 - \alpha_2\alpha_1)BW}}{2}$$

If the eigenvalues of the Jacobian Matrix are real, distinct and opposite, then the system will tend towards the equilibrium point in which both species can coexist. This is the case whenever the radicand is positive and less than $\sigma_1 B + \sigma_2 W$. When this is the case, all values, except those that

lie on the axes, tend to the equilibrium point of coexistence as time goes on, as shown in ***Figure 1*** giving a point of stable equilibrium in which it is very likely that both species will survive.
Figure 1.



This model, however, is a very basic representation of how the butterflies and wasps interact with each other. A more accurate model would show how the population of male and female butterflies are affected by the two different species of wasps that can detect the Anti-aphrodisiac. A model like this proved to be too advanced for our level of understanding in Differential Equations. Also, it is important to note that this model shows what happens in the short run. In the long run, if there is anything that changes in the system, it is possible for one or both species to eventually go towards extinction.

Works Cited

Brannan, James R., and William E. Boyce. *Differential Equations: An Introduction to Modern Methods and Applications, 3rd Edition*. 3rd ed., John Wiley & Sons, 2010.

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