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Executive Summary - Problem C

Introduction

In this problem we attempted to model differential equations describing the interactions between male and female P. brassicae and parasitic wasps Trichogramma brassicae. We also commented on finding the balance between the two populations and the long-term tendencies of development of their interaction.

Biological background

Female butterflies of species **Pieris brassicae** produce a chemical signal to attract males. To prevent the female from having too many suitors, males are able to produce anti-aphrodisiac, which makes the female's aphrodisiac more obscure. The presence of anti-aphrodisiac also increases the odds of successful egg fertilization. The downside is that certain species of parasitic wasps—for example, **Trichogramma brassicae**—can pick up on the anti-aphrodisiac and follow the female to where she lays her eggs and inject their own eggs into the butterfly ones.

Assumptions

- We assume the following values are constant: growth and death factors for both populations, all carrying capacities (maximum sustainable population), percentage of mated female P. brassicae, male-to-female ratio in both populations, amount of anti-aphrodisiac released by each male P. brassicae.
- The probability of being targeted by Trichogramma brassicae is also constant and the same for every female P. brassicae.
- There are no mutations or evolution in the populations studied.

Variables and constants.

- P_1, P_b – population of P. brassicae
- P_2, P_w – population of Trichogramma brassicae
- r_1, r_2 – growth factor of P. brassicae and Trichogramma brassicae respectively
- K_1, K_2 – carrying capacity for P. brassicae and Trichogramma brassicae respectively
- α_{12} – competition coefficient, shows how P. brassicae population's resources are affected by Trichogramma brassicae
- *mf* – percentage of mated female P. brassicae
- M_{aa} – amount of anti-aphrodisiac released by each male P. brassicae
- df_1, df_2 – natural death factors for P. brassicae and Trichogramma brassicae population respectively
- *t* – time
- α_{21} – competition coefficient, shows how Trichogramma brassicae population's resources are affected by P. brassicae

Modeling the interactions with differential equations

As the base for our system of DEs, we chose the Lotka-Volterra model of competition. It not only models the interaction between the two species, but it also takes into account the carrying capacity. The DE for interactions within P. brassicae population should follow the logistic model and take the following into account:

- Percentage of mated female P. brassicae

- Use of anti-aphrodisiac by male P. brassicae
- The effect of wasp population on P. brassicae population
- Natural deaths among the P. brassicae population

Based on these characteristics, we chose this DE to represent the interactions within P. Brassicae population:

$$\frac{\partial P_1}{\partial t} = P_1 * \left(1 - \frac{P_1 + \alpha_{12} * M_{aa} * P_2}{K_1}\right) * mf * M_{aa} - df_1 * P_1$$

The DE for wasp-butterfly interactions should also follow the logistic model, as well as take into account the following:

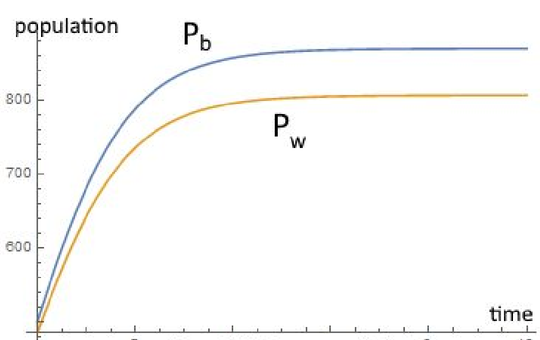
- Wasp population depends on P. brassicae population to reproduce.
- Wasps find female P. brassicae by detecting anti-aphrodisian on them.
- A certain fixed percentage of wasps die each period of time from natural causes.

Based on these characteristics, we chose this DE to represent the wasp-butterfly interaction:

$$\frac{\partial P_2}{\partial t} = P_2 * \left(1 - \frac{P_2 - \alpha_{21} * M_{aa} * P_1}{K_2}\right) * mf * P_b - df_2 * P_2$$

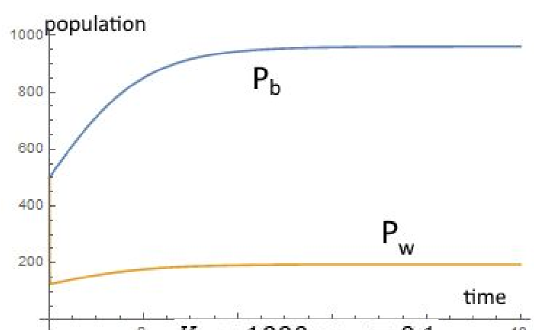
Long run

In the long run, one of the two outcomes is possible: if the competition coefficients are low, then the two populations will reach an equilibrium level near their carrying capacities. However, if one coefficient is much bigger than the other one, then the more competitive species will reach a higher population level before stabilizing:



$K_1 = 1000, \alpha_{12} = 0.1$

$K_2 = 50, \alpha_{21} = 0.1$



$K_1 = 1000, \alpha_{12} = 0.1$

$K_2 = 50, \alpha_{21} = 0.9$

Conclusion

1) The main **trade-off** between the two populations is the amount of anti-aphrodisiac released by male P. brassicae. On one hand, it removes distractions in the form of potential suitors for a mated female butterfly, which allows it to hide its eggs better; it also increases fertility rate, resulting in higher population growth. On the other hand, it lets parasitic wasps find the mated butterflies and infect their eggs, significantly undercutting P. brassicae population.
2) **In the long run**, both populations will eventually stabilize at certain levels. These levels depend on the competition coefficients (CC) — higher CC for predators leads to a high carrying capacity, while a low CC results in a lower carrying capacity. For the prey population, effects are reversed.

References

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