Problem C

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Prompt: Develop a mathematical model to illustrate the interactions of male and female *Pieris Brassicae* with a parasitic wasp species. What is the best balance for this system?

For this problem, we made a number of assumptions. The first and biggest assumption we made was that *Pieris Brassicae* has an infinite life span, and therefore we did not factor a mortality rate into our model. Additionally, we assumed that our population was within a closed system, so apart from the starting population and population reproduced, there would no other butterflies migrating into the model. We decided that the only limiting factor on the butterfly population would be the wasps, which would hijack the butterflies' eggs and thereby prevent reproduction. If a butterfly egg successfully produced a butterfly, the chance of either sex would be 50%. We also assumed that if a male releases anti-aphrodisiac, it would only affect the female he is paired with, not surrounding females. Finally we assume that if a wasp manages to detect anti-aphrodisiacs, it will successfully hijack an egg. Working from these assumptions, we developed two differential equations to model the growth of the butterfly and wasp populations:

$$\frac{dP}{dt} = P\left(R - LW_p\right)$$

$$\frac{dW_p}{dt} = LPW_p - KW_p$$

Where R is a constant describing the chance of a male successfully mating with a female, P is the total population of butterflies at any point, L is a constant describing the chance of a given wasp detecting anti-aphrodisiacs, K is the death rate of the wasps, and W_p is the population of wasps.

These equations do not have solutions that can be expressed simply in terms of time, but based on methods used for Lotka-Volterra[i] equations can be combined to give a solution of the population of butterflies in terms of the population of Wasps.

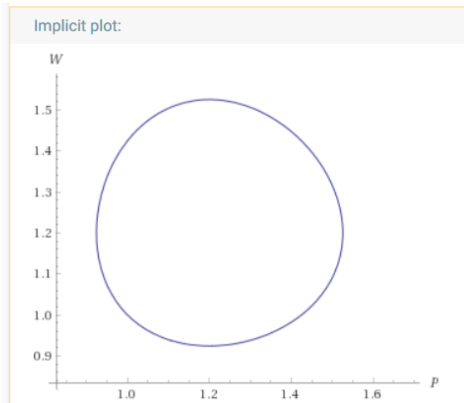
Dividing the two equations gives:

$$\frac{dP}{dW_p} = \frac{P\left(R - LW_p\right)}{LPW_p - KW_p}$$

Separating and integrating gives an implicit solution:

$$LP - Kln(P) = \ Rln\left(W_p\right) - \ LW_p + \ C$$

Where C is a constant of integration. Depending on the values chosen for R, L and K, a graph of this function can yield different results. When we chose the values R=0.6, L=0.5, and K=0.6, the graph becomes circular, where the two populations rise and fall periodically.



By setting dP/dT and $dW_p/dt = 0$, we can find the point of equilibrium where the population of wasps and the population of butterflies no longer change. This is expressed by:

$$\frac{dP}{dt} = RP - lPW_p = 0$$

$$\frac{dW_p}{dt} = LPW_p - KW_p = 0$$

giving two solutions:

$$P = 0\,, \qquad W_p = 0$$

and $$P = \frac{K}{L},\ W_p = \frac{R}{L}$$

Where the first solution represents complete extinction of both species, and the second represents a point of equilibrium, with both populations stable. This point represents the point in which the respective populations are the most balanced.

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[i] "Lotka–Volterra Equations." Wikipedia. Wikimedia Foundation, November 7, 2019. https://en.wikipedia.org/wiki/Lotka–Volterra_equations.