Chemical Espionage
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The evolution of natural ecosystems is dynamic and is based on a number of variable parameters, and thus can be modeled by differential equations.
In this paper, we will take a particular ecosystem, that of the large cabbage butterfly and its parasitic wasps, and we will derive the formulas that link the different parameters of this ecosystem to each other.
To get started, we first need to describe the particular phenomena that we are modelling. Female cabbage butterfly release chemical signals that attract males. In response, males release anti-aphrodisiacs, which mask or dissuade other males.
However, these chemicals have the opposite effect on wasps. Upon detection of the anti-aphrodisiacs, the wasps will follow the scent and lay their own eggs inside the butterfly eggs, thus killing the butterfly offspring in the process.

Using the information given above, we can now talk about the relationships between some of the parameters of this ecosystem. In this paper, we will mainly focus on 3 parameters: the number of butterflies, the number of wasps, and the strength or range of the male chemical signal.

The first model that we are presenting, which is also the simplest and most straight-forward, is one that makes the following assumptions:

- That the probability "a" of a male being in the radius of the female chemical signal is constant.
- That the probability "b" of a wasp being within reach of a male chemical signal is constant.
- That the female butterfly is constantly emitting chemical signals.
- That the only way for wasps to lay eggs is by taking over butterfly nests.

It follows that the rate of change of the number wasps can be modelled by:

$$\frac{dw}{dt} = Z1 * a * b * w$$

Z1 being the wasp's birthrate per nest.
It also follows that the rate of change of the number of butterflies can be modelled by:

$$\frac{dn}{dt} = Z2 * (a*n - a*b*w)$$

Z2 being the butterfly's birthrate per nest.
The solution to these simple differential equations is exponential population growth. For this reason, it is always a more accurate representation of nature to limit the number of animals with a carrying capacity, making the growth a logistical growth that levels off when a maximum is reached.

A big limitation of this model is the fact that the probability is a constant, which is not true. In fact, “a” should depend on “n” and “b” should depend on both “n” and “w”. The probability should be 0 at n = 0, and 1 at n = infinity. Therefore, we can further enhance the model by replacing the constants “a” and “b” by a function of type $-e^{-k*n}+1$.
The resulting equations would give us a coupled system of differential equations which gives us a more realistic but much more complex answer.

The second model that we adopted is one that takes into account signal strength, and how it varies depending on the number of males. The more males there are, the more competitive it become, and the stronger the signal gets. We also assumed in this case that instead of being parasites, the wasps are direct predators of the butterflies. Which means that the wasps don’t need to take over butterfly nests in order to produce offspring. Butterflies are only one of the different preys of the wasps. Therefore, in this case, increasing the number of butterflies is going to increase the carrying capacity for wasps, while increasing the number of wasps is going to have a direct negative effect on the rate of change of the butterflies. The signal strength also increases the wasp’s carrying capacity, as it makes more resources available to the wasp population. Finally, signal strength is directly proportional to the number of butterflies, which stems from the increased competition.

The resulting equations is a very complex and difficult to solve coupled system of equations. Also, it presents many limitations because it makes a lot of baseless assumptions in its formulas and can be replaced by a much simpler system like that of our first model.