Chemical Espionage: SCUDEM IV Problem C

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1 Background

A constant struggle for the *Pieris brassicae*, large white cabbage butterfly, is parasitic wasps that infest the butterfly's eggs with young ones of their own. During the mating season, female *P. brassicae* release a chemical signal to let the male *P. brassicae* know they are ready to mate. In response, the male *P. brassicae* might release an anti-aphrodisiac in order to hide that female's scent from other males who may want to mate with her. While the presence of the anti-aphrodisiac inherently means the male butterflies will be able to fertilize more eggs and the female butterflies will be less bothered by competing males and find better hatching places for their eggs, parasitic wasps can detect the anti-aphrodisiac. When this happens, the wasp will follow the female butterfly who has the chemical anti-aphrodisiac on them and plant their eggs inside the butterfly eggs. We are tasked with finding what the best balance for the system is as well as what is likely to happen in the long run. We start by considering how the interaction between the species and their use of the anti-aphrodisiac affects their populations.

2 Model

When creating our model, we kept certain parameters in mind and made assumptions. In our ordinary differential equation (ODE) model, we assume an equal number of male and female butterflies. We also assume all wasps are female, for simplification, and can insert 20-60 wasp larvae in one butterfly egg. Also if a wasp detects the presence of the anti-aphrodisiac, they will find the butterfly eggs. Other assumptions include: the butterflies are continuously reproducing, there is a carrying capacity for each population, there is a death rate for each population where a percentage of the population dies from their natural life cycle and other reasons. We produce our model accordingly such that $\frac{dW}{dt}$ is the wasp population and $\frac{dB}{dt}$ is the butterfly population.

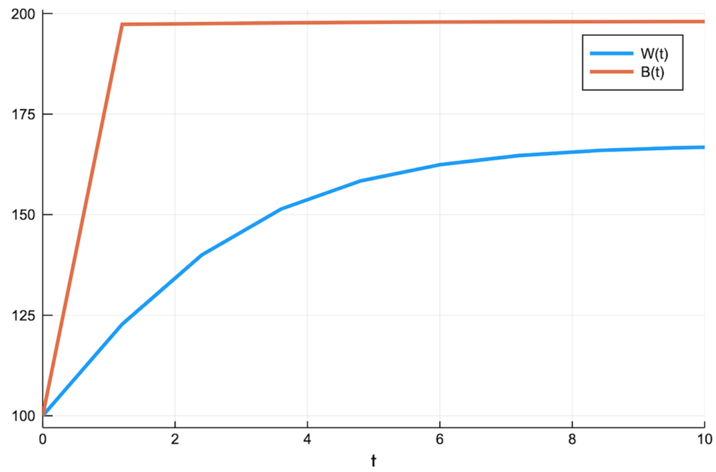
$$\frac{dW}{dt} = W\left(\gamma \cdot \phi \cdot e_b \cdot e_w\right)\left(L_w - W\right) - W \cdot D_w \tag{1}$$

$$\frac{dB}{dt} = -W\left(\gamma \cdot \phi \cdot e_b\right) + B\left(\gamma \cdot e_b\right)\left(L_b - B\right) - B \cdot D_b \tag{2}$$

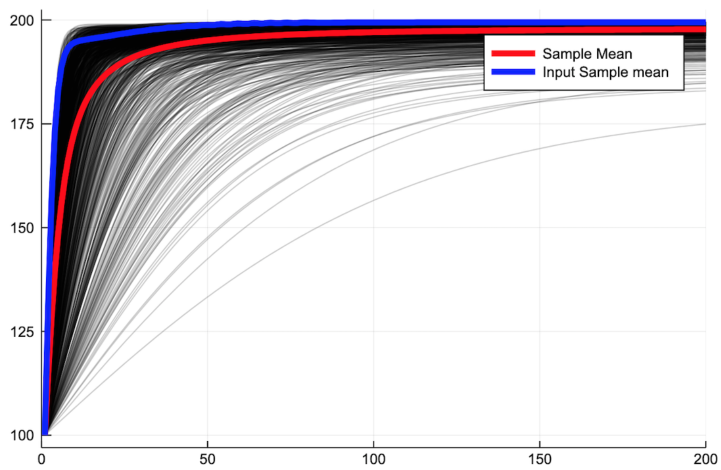
W refers to the wasp population, γ is the interaction between male and female butterflies which determines how many mate at a given time period, and ϕ is the interaction term between the wasps and the anti-aphrodisiacs that are produced. The parameter γ helps determine the interaction between male and female butterflies. This number can represent the proportion of the population that mate at that time point. For ϕ, this parameter is similar to γ but refers to how well the wasps can pick up on the scents produced during the mating at that time period. This value is different since we expect the wasps and butterflies to have different levels of ability to track the scent. Also, e_b is the number of butterfly eggs, e_w is the number of butterfly eggs infected by wasp larvae, L_w is the carrying capacity for the wasps, D_w is the death rate of the wasps, B refers to the butterfly population, L_b is the carrying capacity for the butterflies, and D_b is the death rate of the butterflies.

3 Results

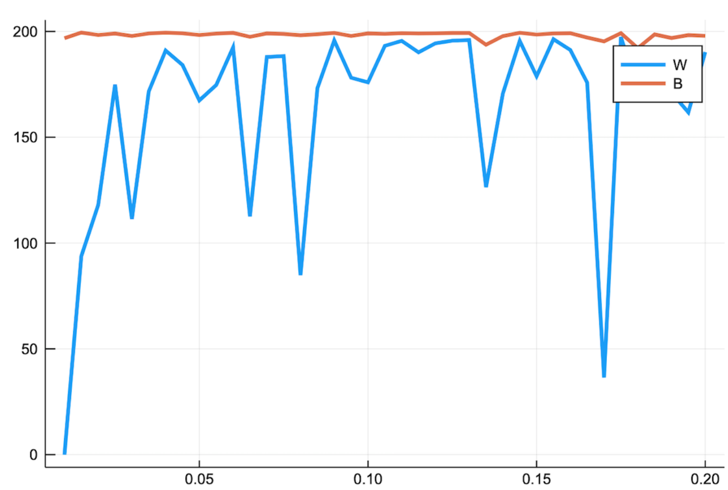
To address the question of what the best balance is for the system, we propose a system in which the butterfly population is higher than the wasp population is most beneficial so the butterflies can keep up with the loss of their one egg to between 20-60 wasp larvae. In the long run, we would expect to see both populations reach an equilibrium where the butterfly population is higher than that of the wasp population.



The graph above shows an example solution for the results of the Wasps and Butterflies. As can be seen in the graph, both populations can survive within the same given space. The rates of the parameters in this example scenario allow for both populations to survive. If the birth rates, death rates, or carrying capacities for either species were to be different, then the output graph will vary accordingly.



The graph above is 1000 simulations of the model for the butterfly population and how it may differ with respect to the random parameters. The sample outputs are in black, the sample input mean is in red and the sample output mean is in blue. It makes sense that the mean and mean output are different because the model is non-linear. A similar sample output can be created with the model about the wasps.



The graph above shows the equilibrium populations of butterflies and wasps for varying values of the parameter ϕ, which is the interaction term between wasps and the strength of the scent from the anti-aphrodisiacs. The stochasticity apparent in the graph is the result of randomly sampling from a Uniform Distribution for the other reproduction-related parameters.