Modeling Chemical Espionage Between *Pieris brassicae* and its parasites, *Trichogramma brassicae* and *Trichogramma evanescens*

Group 02
Coe College
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1 Introduction

Pieris brassicae is a butterfly commonly seen as a pest for its crop attacks on crops in the genus *Brassica*, such as cabbage and cauliflower [1]. Due to this pest behavior, wasps such as *Trichogramma brassicae* and *Trichogramma evanescens* are often introduced to reduce the *Pieris brassicae* population [2]. These wasps reduce the population by mounting a female *Pieris brassicae* and laying parasitic eggs, replacing the *Pieris brassicae* eggs [3]. Male *Pieris brassicae* release an anti-aphrodisiac on females after mating, but this anti-aphrodisiac is also a signal for the wasps [4]. We will model this behavior with a system of differential equations, making some simplifying assumptions.

2 Assumptions

- *Trichogramma brassicae* and *Trichogramma evanescens* can be modeled as one wasp population.
- There is an equal amount of male and female butterflies in the starting population, and the birth rate is 1:1.
- Wasps mounting anti-aphrodisiac-affected butterflies is an innate behavior and not tied to long term memory from previous experiences.
- Both birth rate and natural death rate are constant throughout the year.
- Each female of all three species will lay eggs once through its life-cycle.
- The populations of each species are not affected by extraneous factors other than natural birth/death and symbiotic relationship (in this case, parasitism) between the species.

[1] "*Pieris Brassicae* (Large Cabbage White)." CABI, 11 Oct. 2014, www.cabi.org/isc/datasheet/41157.

[2] Smith, Sandy M. (1996). "Biological control with *Trichogramma*: advances, successes, and potential of their use". Annual Review of Entomology. doi:10.1146/annurev.ento.41.1.375.

[3] Martinus E. Huigens, Jozef B. Woelke, Foteini G. Pashalidou, T. Bukovinszky, Hans M. Smid, Nina E. Fatouros, Chemical espionage on species-specific butterfly anti-aphrodisiacs by hitchhiking *Trichogramma* wasps, Behavioral Ecology, Volume 21, Issue 3, May-June 2010, Pages 470–478, https://doi.org/10.1093/beheco/arq007

[4] see note 3

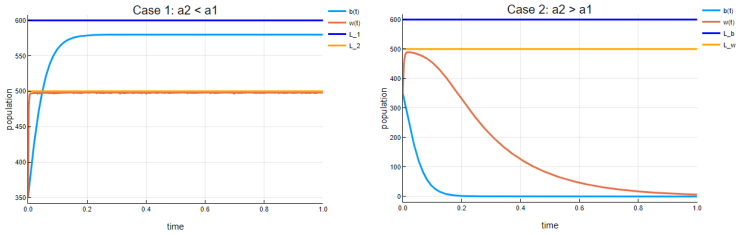
- The wasps are obligate parasites, that is, they must replace *Pieris brassicae* eggs in order to reproduce.
- The parasitism rate is not dependent upon the population of the wasps
- The anti-aphrodisiac is required for a successful mating in *Pieris brassicae* and a successful parasitism by either wasp.
- The eggs of *Pieris brassicae* are replaced by wasp eggs in a 1:1 ratio.

3 Model

We modeled the populations using the following system of differential equations:

$$\frac{db}{dt} = -d_b b + (a_1 - a_2) e s_b b (L_b - b)$$
$$\frac{dw}{dt} = -d_w w + a_2 e s_w b (L_w - w)$$

Where d_b is the death rate of the butterflies, d_w is the death rate of the wasps, a_1 is the factor of the successful pairings due to the anti-aphrodisiac, a_2 is the factor of the parasitism rate due to the anti-aphrodisiac, e is the number of eggs laid by *Pieris brassicae*, s_b is the fraction of butterfly eggs that go on to mate, s_w is the fraction of wasp eggs that go on to mate, L_b is the butterfly carrying capacity, and L_w is the wasp carrying capacity.



(a) $a_2 < a_1$ (b) $a_2 \geq a_1$

Figure 1: In the first case, the benefits of the anti-aphrodisiac outweight the costs to *Pieris brassicae*, and thus both butterflies and wasps reach a sustained population close to their carrying capacity. In the second case, the costs of the anti-aphrodisiac limit the population of *Pieris brassicae*, which then limits the population of the wasps, as they are obligate parasites

The above figure illustrates the two qualitative cases for the output of this model. The number of successful pairings is limited to 0.5 times the population ($a_1 \leq 0.5$), but the parasitism rates of these wasps has been reported at $50\% - 100\%$ ($a_2 = 0.5 - 1.0$)[5], so the second case is more likely. The first system allows both butterflies and wasps to reach a population near their carrying capacities, and thus is the more balanced option.

[5]Van Heiningen TG, Pak GA, Hassan SA, van Lenteren JC. 1985. Fouryear's results of experimental releases of *Trichogramma* egg parasitesagainst lepidopteran pests in cabbage. MedFac Landbouww Rijksu-niv Gent. 50:379–388.