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Executive Summary

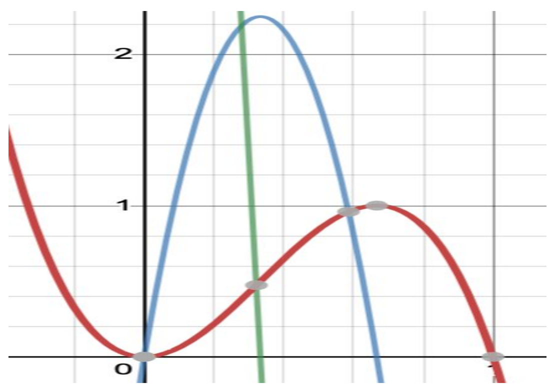
Chemical signals are a potent way to attract potential mates to have offspring and keep the genetic line going. Many creatures in the animal kingdom exhibit this trait. We will dive into the complexities that follow this ability for the Large Cabbage White Butterfly (Pieris Brassicae)(LCWB).

LCWB females secrete an aphrodisiac detectable by males. This then attracts males to mate with the females. Although this would typically only be beneficial, the large quantity of males attracted to aphrodisiac-secreting females would actually pose one major problem. The inherent desire for each male to be the one who mate means that the female will be stressed, as she is being distracted by many males, resulting in an adequate time investment in laying eggs in a safe place, leaving them prone to many hungry common predators.

The males of the species have developed an anti-aphrodisiac (AA) chemical which masks the output of the female, limiting the number of males, an ideal state, yet wasps are (literally) on their scent. Multiple species of parasitic wasps are able to identify the AA, using it as a beacon for locating the female, regardless of the egg's advantageous location. The wasps then lay their larvae on top of the LCWB eggs, which proceed to eat them before they have the chance to hatch into caterpillars. So with AA helping solve a problem, while providing another, how can the LCWB survive? We were asked to determine a balance between these two competing interests.

In these two predators, we see differing methods of butterfly termination. Knowing that the average clutch size is 40 eggs, 80 per year (assuming the average two broods per year), it becomes clear that the parasitic wasps are a much more potent predator through their total eradication of a clutch at once. The common predator, on the other hand, catches and kills butterflies one-by-one, yet at a higher rate, the less advantageous the eggs' location.

As a final premise, it must be stated that we are comparing the scenario of average population outcomes per year per female on the basis that the most pertinent factor for stunting population growth is the use of AA. This is modeled in the graph of AA use by males vs. average surviving population per female per year, once more with value 1 in the y direction being the absolute maximum success of a LCWB female's eggs surviving. The x direction is from 0 to 1 where 1 means all males partake in AA use.



The equation to express our model is the following:

$$r = 6.75(x^2 - x^3) \text{ (in red)}$$

$$0 \le x \le 1,\ 0 \le r \le 1.$$

Where x is AA used (from none at x=0 to all males using AA x=1), and y being safety (0 being totally unsafe, and 1 being maximum safety but not necessarily the entire population surviving)

This graphed has a simple behavior; a gentle rise from 0 to 1, then a sharp drop from that peak down to 0 at x=1.

There are several reasons we chose this as our main equation to describe the behavior of predatory effects on the population of the LCWB. Firstly, the graph begins at (0,0), meaning there would be too many predators killing the butterflies since they would likely be found in large groups of males attracted to females, making them easy prey for common predators. The danger from wasps is virtually 0 at x=0. Then, as more portions of males use AA, less and less males would heard around the female, making it easier for her to hide her clutch of eggs from the common predators, but allowing for vulnerability to parasitic wasps. Since less males using AA are harder to detect by the parasitic wasps, the wasp's effect on controlling the population is lesser when specific portions of males are using the AA.

$r' = 6.75(2x - 3x^2)$ (in blue)

The derivative tells us more about what we see in the trend. The r' graph is a negative parabolic function that crosses positive from x= 0 to approximately 0.333. At 0.333, r'' is 0, meaning the slope in the r graph is still growing but at a decreasing rate. This means the effect of the wasp's predatory behavior is having an effect on the trajectory of the graph. From there, at 0.667, The maxima of the r graph is achieved, meaning the danger from both predators is equal, as in, general predators are killing as many butterflies as parasitic wasps kill butterfly eggs, so since the danger from these predators is equal, and are both exponential in nature, then it is at this point in which the butterflies are in minimal danger from both predatory groups.

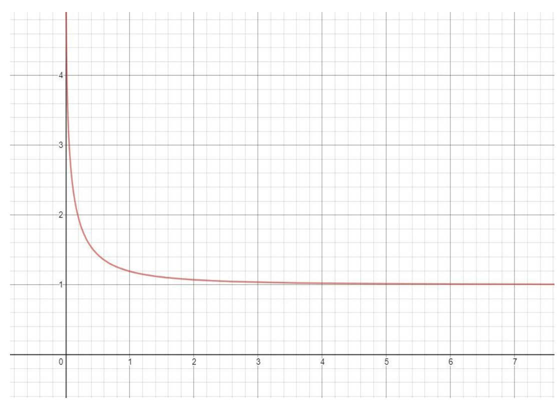
In determining a model for the male to female relationships within the LCWB population, it became clear that our safety equation was becoming increasingly hard to ground in real life. This, then, became a prime opportunity to look to a real-life situation in which the LCWB's interactions with parasitic wasps and the common predator is actualized. Here, we analyzed the eradication of the LCWB population seen in Nelson, New Zealand in late 2014. Nelson declared the LCWB a pest in 2010 and called for its eradication, due to the tendency for its larvae to completely eat the leaf when nearing adulthood. Nelson's Department of Conservation declared the pest eradicated in December of 2014, after releasing the same parasitic wasps studied in our model, along with placing $10 bounty on each butterfly, resulting in the capture of 134 butterflies.

The equation to express the current population of the LCWB is as follows: population=(decay)y'-external factors, which when adapted to the Nelson study, can see a population of 0 as a result of the same pressures modelled above, with the external factor primarily being human capture, therefore the model is as follows:

$$: \ 0 = (6.75x^2 - 6.75x^3)y' - 134$$

This equation results in a y of 19.85 $[ln(x) - \frac{1}{x} - ln(x-1)] + 1$. The constant multiple, 19.85 purely affects the speed at which the graph approaches its asymptote at 1 (achieved assuming the constant of integration is one), and thus when looking to a general solution applicable to any scenario, this constant multiple can be made K, for simplicity. Finally, this form of the function, $y = K[ln(x) - \frac{1}{x} - ln(x-1)] + 1$ is bounded by x=1, thus it can be parameterized for reader's clarity, assuming the final form of:

$$y = [ln(x+1) - \frac{1}{x+1} - ln(x)] + 1$$



Ths y function, marked in red, represents the relationship between males and females in the LCWB population, as with increasing AA, we can see that the function's outputs approach one, ultimately becoming one after x=32, translating to only one male bothering/mating with the female. The constant of integration, 1, is simply the assumed minimum number of mates per female, so in our case, we assumed that at least one male would mate with every female. Under this model, the output of the function can be interpreted as an average male number per brood, 1.5 meaning a fifty/fifty chance to mate with one male or two males.

As a result of our research, we have concluded that the most accurate model for this species of butterfly, and more generally, for the intricate balance of forces that affect a population is by maintaining a simpler model that is indicative of many different factors that come into effect. The way we achieved that with our efforts is by using a case study to extract data on the decay rates of the butterflies when many of the highly potent predators, parasitic wasps, are introduced. The data was then applied to the simple counteracting exponential functions that show the male portions using AA vs. the safety of the population of butterflies from predators. This showed us a balance point Where they are safest from such creatures. In the long term, these populations will vary in a virtually unpredictable manner unless outside forces intervene, they may behave oscillatory, from high populations to an extinction.

In retrospect, we realized we did not find explicit equations that may give us long term trends of populations, though our equations may be applied to get a survivability rate of these species. The best type of equation for that is likely a function based in Euler's number, e, for long term exponential growth and decay, and due to time constraints and difficulty of finding information, we did not conjure up an eloquent equation that does that job. We believe this to be due assumptions to fill the information gaps would be too crude, yet further research into case studies such as Nelson may have provided us with this knowledge. Furthermore, our general equation, and thus our Nelson equation, ignore a very potent factor: pesticides. Human use of pesticides is rampant in many countries across the globe and upon reflection, it is clear that pesticides would be even more potent as a decay than our constant additive allows for. This though, is not dependent on anti-aphrodisiac thus escaped our model's breadth of coverage, yet pesticides still should have been factored into the Nelson equation in increasing that 134. Lastly, we would have liked to produce a differential equation that was not separable, and thus had to be attacked from multiple angles, utilizing knowledge from SIMIODE's technique narratives, yet the desire for simplicity and readability won out.

References

https://www.simiode.org/resources/techniquenarratives

https://www.stuff.co.nz/environment/86754492/successful-eradication-of-great-white-butterfly-in-nelson-world-first

For all graphs included - https://www.desmos.com/calculator