Math Modeling Competition
Question #3 Executive Summary

Introduction

Our model displays the relationship between *Pieris Brassicae* butterfly, *Pieris Brassicae* eggs and *Trichogramma* wasps. We considered *Pieris Brassicae* eggs to be a separate population, as they are vulnerable to hijacking from the wasps prior to hatching. Our model seeks to find the influence of anti-aphrodisiac production on the long term behavior of these three populations.

Analysis

The model of this problem examines the effect of anti-aphrodisiac production in female P. Brassicae on three populations. Without the anti-aphrodisiac, females are likely to be bothered and place eggs in a less than ideal location. However, meddling wasps will hijack the eggs increasing proportional to the amount of anti-aphrodesiac produced. In order to provide some structure to our model, the following assumptions were made:

1. We consider the three populations in isolation, *P.Brassicae*, *P.Brassicae* eggs, and *Trichogramma* wasps.
2. Butterflies lay eggs proportional to their population.
3. Both butterflies and wasps have death rate proportional to their population.
4. Wasps can only lay their eggs in *P.Brassicae* eggs.
5. Male butterflies will bother the females during the egg laying process causing the eggs to be suboptimal placed. The bothering will be a function of the number of males, the number of eggs and the amount of anti-aphrodisiac produced.
6. The wasps will hijack the eggs and kill them. his hijacking will be a function of the number of eggs and the amount of anti-aphrodisiac.
7. Eggs that are unbothered and un-hijacked become butterflies at a constant rate.
8. Eggs are laid in batches of 50
9. Butterflies live about twice as long as wasps
10. The ratio of male to female butterflies is approximately constant

With these assumptions made we produced the following differential equations:

$$\frac{dB}{dt} = \beta E - \delta B \ , \ \frac{dE}{dt} = \alpha B - c_1 \frac{B^2 E}{1+A} - WE(c_2 A + c_4) - \beta E \ \text{ and } \ \frac{dW}{dt} = W(E(c_3 A + c_5) - \gamma)$$

Where our parameters are as follows:

1. α = Egg production rate
2. β = Egg hatch rate
3. γ = Wasp death rate
4. δ = Butterfly death rate
5. c_1 = proportionality of bothering
6. c_2 = proportionality of wasp hijacking due to anti-aphrodisiac
7. c_3 = proportionality of wasp hijacking due to random chance
8. c_4 = proportionality of wasp birth due to anti-aphrodisiac hijacking
9. c_5 = proportionality of wasp birth due to random hijacking
10. A = amount of anti-aphrodesiac

We solve for the equilibrium solutions of the system of differential equations using a numerical solver in matlab. We find three equilibrium solutions, one in which all populations die out, one in which only butterflies and butterfly eggs remain, and one in which all three populations have a positive long-term value. Figure 1 shows the outputs from our solutions with varied amounts of

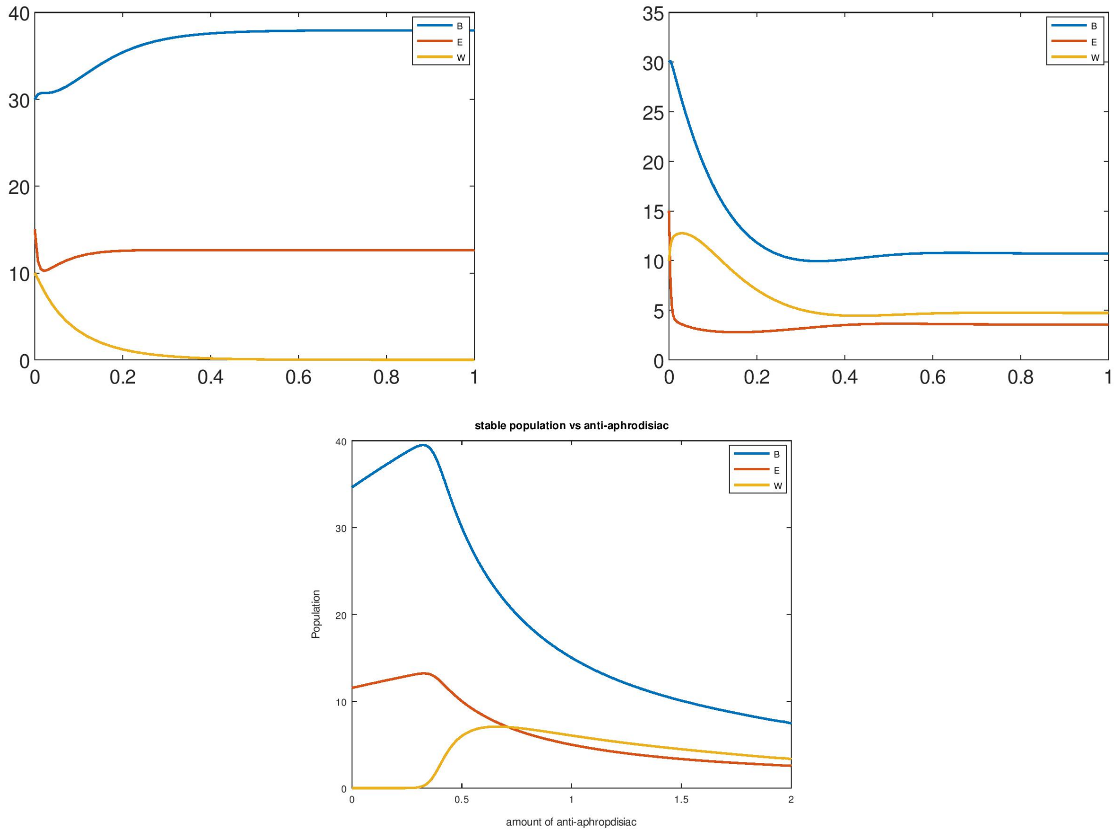

Figure 1: The top two plots display snap shots for specific values of A in which our non-trivial equilibrium solutions are displayed. The first of which involves too little anti-aphrodisiac for the wasps to survive, and the second shows a stable long-term population reached for all three groups. The final is described in our conclusion.

anti-aphrodesiac.

Conclusion

We conclude that at a low level of anti-aphrodisiac, the decrease in bothering outweighs the attraction of wasps, allowing the butterfly population to increase as anti-aphrodisiac increases. However, at higher levels of anti-aphrodisiac, the attraction of wasps outweighs the decrease in bothering, hurting the butterfly population. The ideal balance point, from the standpoint of biodiversity, can be seen in the bottom graph of anti-aphrodisiac and population at an anti-aphrodisiac value of .5. At this value, the system has the best biodiversity. At higher values, populations of both wasps and butterflies begin to suffer and at lower values the wasp population begins to die off.

References

"Chemical espionage on species-specific butterfly anti-aphrodisiacs by hitchhiking *Trichogramma* wasps, "Martinus E. Huigens, Jozef B. Woelke, Foteini G. Pashalidou, T. Bukovinszky, Hans M. Smid, and Nina E. Fatouros.*Behavioral Ecology*. Volume 21, Issue 3, May-June 2010, Pages 470–478, 11 February 2010. https://doi.org/10.1093/beheco/arq007.

How to Manage Pests. (n.d.). Retrieved November 5, 2019, from http://ipm.ucanr.edu/PMG/NE/trichogramma_spp.html.

Rothschild, M., Valadon, G., & Mummery, R. (2009). Carotenoids of the pupae of the Large White butterfly (Pieris brassicae) and the Small White butterfly (Pieris rapae). *Journal of Zoology*, *181*(3), 323–339. doi: 10.1111/j.1469-7998.1977.tb03247.x