**Executive Summary**

2019 SCUDEM Competition


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**Problem C: Chemical Espionage**

## Introduction

The purpose of this model is to show the relationship between the population of the large cabbage white butterfly Pieris brassicae and parasitic wasps. A male Pieris brassicae is using chemical signals, called anti-aphrodisiacs, to mask or dissuade other males so that the butterfly can fertilize the eggs. The trade-off of using this chemical, on the other hand, is that there are parasitic wasps which can exploit the chemicals and lay down their eggs into the eggs of the butterflies, which will be eaten by the larvaes then. So, this model shows how the amount of anti-aphrodisiacs used affects both populations and leads to the growth or decline of the populations.

## Variables

$N_B$ ~ Butterfly population at time t
$N_W$ ~ Wasps population at time t
t ~ Time
$r_B$ ~ Growth rate of butterfly population
$r_W$ ~ Growth rate of wasp population
$b_B$ ~ Birthrate of the butterfly (number of eggs that the butterfly can lay per period)
$d_B$ ~ Mortality rate of the butterfly (percentage of the old population will die, range)
$b_W$ ~ Birthrate of the wasp (number of eggs that the wasp can lay per period)
$d_W$ ~ Mortality rate of the wasp (percentage of the old population will die, range)
p ~ The probability that the wasps can successfully find the butterfly's eggs, ($0 \leq p \leq 1$)

## Assumptions

For our model, we made several assumptions, which are stated in the following. Since we think that the male butterflies are utility-maximization beings, they are not very considerate of side-effects and act beneficially for themselves so that they release enough anti-aphrodisiacs to avoid all other mating competitors. In other words, they are not too rational to think of the natural selection of their kinds over the needs to mate. The parasitic wasps can only lay eggs inside the eggs of the white cabbage butterfly, meaning that it cannot lay eggs at all if it cannot find the butterfly eggs. Also, we assumed that the food resources for both populations are infinite and that there is no biological boundary regarding space. To keep it simple, we are just going to focus on our variables, even though there might be other effects, such as that the wasps can lay eggs into eggs of different species. These additional excluded parameters are assumed to be constant.

## Model

$N_B(t) = N_{B0} \cdot e^{r_B t}$ with $r_B = b_B(1-p) - d_B$

$N_W(t) = N_{W0} \cdot e^{r_W t}$ with $r_W = p b_W - d_W$

Notice that $p = f(N_B, N_W)$

## Analysis

First of all, we begin by looking at the growth of the butterfly population without the intervention of the wasps. Ideally, it should increase at an exponential rate, meaning $r_B$ remains a positive constant (or fluctuate slightly) through time.

However, in nature, there is a probability that the parasitic wasps can be attracted by the chemical anti-aphrodisiacs, and successfully trace to the butterfly egg. We use variable p to represent this phenomenon. Then, (1-p) turns out to be the probability that a butterfly egg can survive from the parasite on average. Also, given our assumption about the way the wasps can lay eggs, p implies the probability that a wasp egg can be laid on average. In the model, p is used to explain the growth rate of the population of two species.

$$r_B = b_B(1-p) - d_B$$

$$r_W = pb_W - d_W$$

By looking at the sign of the partial derivative, we can conclude that there is a positive correlation between p and $r_W$, and a negative correlation between p and $r_B$.

$$\frac{\partial r_w}{\partial p} > 0 \text{ and } \frac{\partial r_b}{\partial p} < 0$$

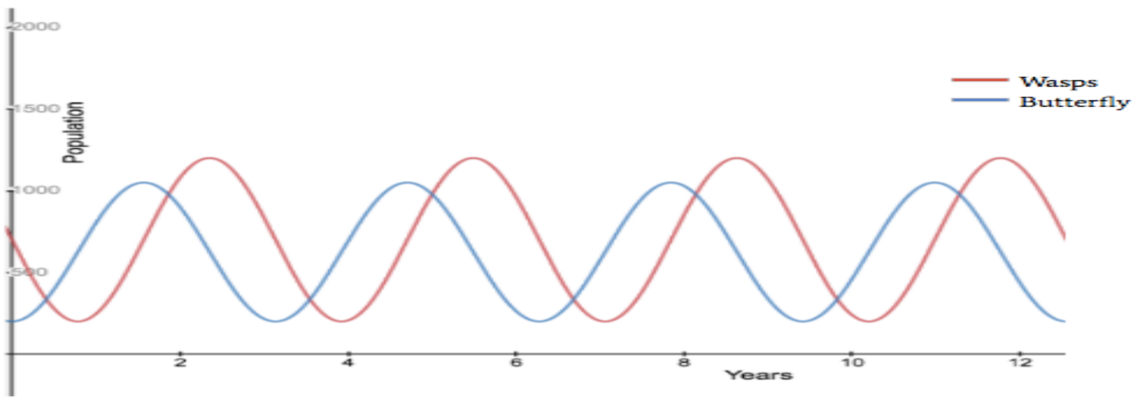
Moreover, p also happens to be a function of $N_B$ and $N_W$, in which p shares a positive correlation with both variables. It is reasonable to argue that as the butterfly population increases, the amount of anti-aphrodisiacs as well as the butterfly's eggs increase, making it easier for the wasps to find the butterfly's eggs, thus increases p. Similarly, when the population of wasps increases, it is harder for the eggs of the butterfly to survive from parasitism, which increases p, and decrease (1-p).

$$\frac{\partial p}{\partial N_b} > 0 \text{ and } \frac{\partial p}{\partial N_w} > 0$$

In the short term, let assume that the population of the wasp and the butterfly start at a moderate level. Thus, p also starts at a moderate level (range from 0.3 to 0.5 approximately). At this level, $b_B(1-p) > d_B$ and $pb_W > d_W$, so the growth of two populations are both positive. Therefore, as time increases, the population of the wasp and the butterfly keep getting larger. However, when the population of the butterfly increases, we know that p increase accordingly, leading $r_B$ and $r_W$ to decrease and increase respectively. As a result, in the first period, $N_B$ increases at a decreasing rate, and $N_W$ increases at an increasing rate.

In the long term, we can see from the graph below that there are many turning points on the lines of two populations. These are the cases in which p reaches its extreme ends: close to 1 and close to 0. Continuing the short term story, when $N_B$ and $N_W$ increases to certain points, p becomes large enough to make $b_B(1-p) < d_B$, resulting in a negative $r_B$. After that, $N_B$ faces downward sloping trend, while $N_W$ still increases but at a decreasing rate. More importantly, p is still increasing slightly because the increase in $N_W$ are more significant than $N_B$ at this stage. However, not long after that, $N_B$ become small enough to become a dominant factor affecting p. This leads to a decrease in p. Thus, $N_B$ starts to recover, while $N_W$ will fall due to the lack of resources to lay eggs in the environment.

The produces explained above will repeat as time increases, making the two populations fluctuates accordingly. The graph for two populations is demonstrated below:




**Limitation**

Our model cannot determine an actual formula showing the relationship between p and $N_B$ and $N_W$. Also, we believe that in nature, the amount of anti-aphrodisiacs is not fixed. The male butterfly only uses anti-aphrodisiacs under high pressure, in which the population of the male butterfly is too large, making the mating process becomes competitive.

**Reference**


Beals, M., Gross, L. and Harrell, S. (1999). PREDATOR-PREY DYNAMICS:LOTKA-VOLTERRA. Available at: http://www.tiem.utk.edu/~gross/bioed/bealsmodules/predator-prey.html [Accessed 9 Nov. 2019].