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Executive Summary
Problem C: Chemical Espionage

When it comes to procreation, insects have a wide array of habits. One common method the female insects use is the secretion of chemical signals to attract a male. The male, once it has found the female will secrete another chemical signal known as an anti-aphrodisiac to ward off competition from other males. A system is then created between the males and females, and an example of this will be explored using a butterfly species, *P. brassicae*. This interaction becomes a dynamic system when a parasite-host interaction is considered. The parasite in this case is two species of wasps, *T. brassicae* and *T. evanescence*, that lay eggs inside the butterflies egg, thereby killing off the host egg. The wasps are able to locate the butterfly eggs through a sophisticated espionage-and-ride strategy where they detect the anti-aphrodisiac signal given off by the male butterflies, which indicates a successful breeding. Since they are so small, the wasps ride on the butterflies to where they will lay their eggs. This dynamic community system was explored in depth, and an attempt to create a mathematical model for it was done; emphasizing what the best balance for the system will look like, and how the system will behave in the long run.

In order to create a model for these interactions, several factors needed to be defined. First, the lifespan of the insects, how many eggs they produce per clutch, and how many clutches they can produce in their lifespan were discovered. After doing background research the conclusion was made that both wasp species take an average of 5 days to hatch from their eggs, after which they have approximately 2 days to lay up to 2 clutches of eggs containing roughly 60 eggs each. *P. brassicae* butterflies lay an average 35 eggs per clutch with there being two generations per year. These parameters were used to derive a simple preliminary model that leaves room to create a more accurate model of this dynamic system.

In developing this model many assumptions were made in order to simplify the parameters that were in the system, and are as follows: No butterfly egg can be completely hidden from the wasps, i.e. probability of being found is never zero. Wasps and butterflies can only die due to natural causes or as eggs. The interaction between the male and female butterflies does not affect the butterfly reproduction rate. The death rate of the wasps is 1 wasp for every 7 days. The death rate of the butterflies is 1 butterfly for every 180 days. The wasps lay 60 eggs per clutch. The butterflies lay 35 eggs per clutch. The space in which the system occurs is infinite and bounded to the first octant. The wasps' only source of a host is the butterfly species. Only these three distinct insects exist in the system, i.e. there is no interaction between the system and the surrounding environment. The system

is homogenous, i.e. the wasps and butterflies are evenly dispersed in space so that there is a constant probability that a wasp will detect a nearby mated female butterfly. The combined signal from the anti-aphrodisiac from the male after mating is what attracts the wasps. A wasp will only infect a single butterfly egg when it lays its eggs. The wasps will not attack the butterfly larvae, i.e. once the butterfly hatches it is safe. All wasps eggs will grow into adulthood. All uninfected butterfly eggs will grow into adulthood. Only one adult will be produced per egg for both species.

Given these assumptions, a simplified model was formulated. This model shows how the population of both species behave with respect to each other. Many terms within it are defined quantitatively as constants, and they include the reproduction rate of the wasps and butterflies, i.e. the birth rate and/or the number of eggs each of them produce per day, the growth rate of the butterfly eggs, i.e. the rate at which a butterfly egg grows into a state where the wasps can no longer harm it, and the death rate of both insects per day. The values of these were estimated given results of previous field studies to be as follows: The birth rate of butterflies is (35/365)days^{-1} meaning that for each successful mating event, a butterfly will produce 35 eggs in the timespan of 365 days. The birth rate of the wasps was estimated to be $\frac{60\cdot(0.75)^2}{2}\ days^{-1}$ meaning that for each successful laying of eggs the wasps will lay a clutch of 60 eggs. It is more complicated though because the background study shows that a wasp has on average a 75 percent chance of laying that clutch of eggs and that it can lay up to 2 clutches of eggs in its lifetime, meaning that the probability of a wasp laying 2 clutches of eggs is 0.5625. The wasps also only have an adult lifespan of 2 days, thus the constant number of eggs produced is divided by the days they do it in to result in units of days^{-1}. The growth rate of the butterfly eggs is approximated to 1/10 days^{-1} meaning it takes 10 days for the butterfly eggs to reach a safe state after they are laid. The death rate of the butterflies is 1/180 days^{-1}, and the death rate of the wasps is 1/7 days^{-1}.

The remaining constant in the model is the death rate of the butterfly eggs due to a parasitism event, i.e. a wasp successfully laying its eggs. This is the constant that the model focuses on to determine what the best balance for the system is, as well as what will eventually happen to the system, at least mathematically. In the end, the model depicts the system as an oscillation that will either amplify or most likely dampen causing the death of insect populations and a collapse of the system. Biologically what this means is not the complete extinction of the insects, but rather a new dynamical system will be created in which the survival of the butterflies no longer depends on the reproduction of the wasps. It is explained exhaustively with the current scientific data supporting the theory of evolution. This means that the butterflies that secrete the anti-aphrodisiacs will eventually die off, and the ones that genetically mutate to not use the chemical signals will become more prevalent. This does not necessarily imply that the butterflies will stop the use of chemical signals entirely, but rather they will not make use of any that attract the wasps species, thus eliminating the system this model is describing.