Matthew Martin, Nathan Bodenman, Timothy Hartley
George Fox University Team 1

Executive Summary

Our group chose the third option, modeling a population of butterflies and parasitic wasps, for this modeling activity. In the initial summary of the population, we are given details regarding a butterfly population where there are interactions between the male and female butterflies that both help and hinder the growth and sustainability of a population with a parasitic wasp that takes advantage of the butterflies reproduction interactions.

Equations:

Our group decided to split the butterfly population into males and females because though they are from the same species, the interactions between them and the wasp population we felt warranted separate equations. The three equations we came up with are:

$$M'=FM(C_M-M)-(MW-MF)$$
$$F'=FM(C_F-F)-(FW-MF)$$
$$W'=W(C_W-W)+MW+FW$$

These equations include three variables: M, F, and W and only one explicitly written constant C_x. M stands for the male butterflies, F stands for the female butterflies, and W stands for wasps. The interactions between the species or genders is modeled by multiplying variables by each other. For example, in the wasp equation, +MW stands for the positive relationship, for the wasps, between the male butterflies and the wasps. Although not explicitly written into the equations, we know that there are constants in front of each term of all the equations. However, without starting information such as carrying capacity for each of the species, how aggressive the wasp larva are, and the ratio of male to female butterflies, we chose to leave all the constants unwritten except C because this constant determines how large any of the populations can become because the environment does not have unlimited resources and we can not have an infinite population of butterflies or wasps.

To develop these equations, we had to make several assumptions about the butterfly-wasp system. Firstly, the male and female populations only increase due to an interaction between the male and female of the species. This is modelled by a component of F*M in the M' and F' equations. We also assumed that there is a population cap. This was imposed onto both genders of butterflies and the wasps. We are making the assumption that the wasps rely primarily on the butterfly eggs as a food source to survive and that the butterflies have a max population that is limited by the environment they live in. Currently, our wasp equation is set up in such a way that the wasps can survive even if the butterflies all die out. While this may not be true if the wasps rely solely on the butterflies as a food source for their larva, the wasps may also have the ability to adapt to find an alternate food source if the butterflies are eradicated from this population.

The population caps or carrying capacities are represented as a variable for the population cap (C_M or C_F) minus the current population (M or F). Next, we assumed that negative change to the population will come from interaction with wasps. To account for the negative effect that the wasps have on the male population of butterflies, a component of the male population multiplied by the wasp population is subtracted from the growth of the males. This represents the wasps infiltrating part of the eggs and stunting male butterfly population growth. It is important to note that the wasps are not directly eating the butterflies, rather the wasp larva are eating the butterfly eggs before they hatch. As a slight simplification, the equations are set up more similar to predator prey models where the predators directly ate the prey. This makes it easier to model and understand because the adult butterflies can have multiple batches of eggs, the wasp larva might only eat some of the butterfly eggs, and other nuances that get very complicated quickly.

With these equations, we were able to calculate some equilibrium points. Firstly, the point (0, 0, 0) is an equilibrium, as should be expected for a population model. There is also an equilibrium point at (0, 0, C_W). This is due to our assumption that without the butterfly eggs, the wasps will find other ways to reproduce and will not simply die off. Without more initial or known conditions, we do not know if this system will have any other equilibrium points or if the system will always lead to an eradication of the butterflies.