SCUDEM IV
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Problem C: Chemical Espionage

Executive Summary

We considered a system with a population of butterflies P_B with an assumed birth rate b, and a population of Wasps P_W with an assumed death rate m. In the butterfly population, it was assumed that male butterflies consisted half of the total butterfly population, $P_{B,male} = \frac{1}{2}P_B$. The male butterflies release a chemical deterrent when mating with the female. Once complete, the female will generate a surrounding deterrent field of concentration c, which we assume to be radially symmetric and constant for the egg laying period. The concentration gradient of the deterrent chemical, in polar coordinates, for an individual i was assumed to take the form,

$$c_i = \frac{\gamma B_i}{r_i^2} \quad (1)$$

Where γ is a proportionality constant and B_i is the amount of deterrent on the female and for simplicity we assumed $B_j = \gamma = 1$, which will be used later. For the population of butterflies, the total concentration of deterrent at some distance r_o from the origin of an imposed coordinate is,

$$c(r_o) = \sum_{i=1}^{\frac{P_B}{2}} \frac{\gamma B_i}{(r_o - r_i)^2} \quad (2)$$

For ease of calculation, $r_j = r_o - r_i$, $B_i \to B_j$, $\therefore c(r_o) \to c(r)$. In the wasp population, they have a sensitivity to the chemical deterrent, defined as K_W. It is also assumed that the wasps take the optimal trajectory along the deterrent concentration gradient. We then modified the Lotka-Volterra predator-prey models with a simple model of chemical diffusion for the butterfly population:

$$\frac{\partial P_B}{\partial t} = bD\nabla^2 c - P_W K_w D\nabla^2 c = (b - P_W K_W) D\nabla^2 c = 4D\gamma(b - P_W K_W)\sum_{j=1}^{\frac{P_B}{2}} \frac{B_j}{r_j^4} \quad (3)$$

Where D is the diffusivity of the deterrent. Furthermore, we assumed an average dispersion radius R between all butterflies for simplicity. In this case, $\partial \rightarrow d$ for the time derivative because we assume a relatively stationary butterfly during the egg laying period, thus $R \neq R(t)$. This simplifies our equation to:

$$\frac{dP_B}{dt} = \frac{2D}{R^4}(b - P_W K_W) P_B \quad (4)$$

$$\frac{dP_W}{dt} = (-m + P_B K_W) P_W \quad (5)$$

We used these equations in our preliminary numerical solutions for the butterfly and wasp populations.

Ultimately, our findings with these models were that the wasp population is very unstable, and it is wildly affected by varying sensitivities. Regardless of the numerical values chosen for the parameters, the wasp population will tend to rise to a peak then fall and stabilize at a relatively low amount, often lower than that of the butterfly population. On the other hand, the butterfly population is relatively stable and will hover around its initial population amount regardless of parameter values. We believe it is beneficial for the butterflies to stay far apart and for the wasps to closely regulate their sensitives.