Executive Summary

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Statement of Problem: What is the best balance of the system of interactions between the male and female P. brassicae as well as the parasitic wasps?

This system is based on the various interactions between the male and female large Cabbage white P. brassicae butterflies when aphrodisiacs are involved as well as parasitic wasps that use this same chemical reaction to track mated females. The idea between the trade-offs of this chemical use is important as the males respond to the aphrodisiac by mating with the female then releasing an anti-aphrodisiac in order to keep other males away from the mated female. While this anti-aphrodisiac may keep other male P. brassicae away, it also unintentionally attracts the parasitic wasps that use this chemical mix to lay its eggs in the freshly laid butterfly eggs. Using this system, we made several assumptions to build this model:

(1) The typical female White Cabbage Butterfly releases some necessary amount of aphrodisiac to attract males.
(2) Both the wasp and butterfly depend on each other and there is no other interactor in the system.
(3) Once a Male releases the anti-aphrodisiac, no other males mate with the female.
(4) Either all the eggs are eaten or all the eggs survive.

In order to model this system of interactions between the male and female butterflies along with the wasps, we decided to base off our model from the lotka-volterra standard prey-predator model.

This model that we were able to develop designs the system as a homogenous mixture. In this system, there is the presence of both the wasps and butterflies along with the chemical mixture of the aphrodisiac. The set of equations that we designed are as follows:

$$\frac{db}{dt} = b(g - k_1 Cw)$$

$$\frac{dw}{dt} = w(k_2 Cb - y)$$

$$C = k_b \frac{1}{b} - k_w \frac{1}{w}$$

g: growth rate of butterflies
$k_1 C$: predation rate factor
w: wasp density
$k_2 C$: Predator success factor
y: Death rate of wasps in absence of host
C: concentration of anti-aphrodisiac

In this model, the equations used are dependent on each other to represent the relationship between the wasps and butterflies. Both also depend on the concentration of antiaphrodisiac. This first equation represents the change in the density of the butterflies over time. This is shown by the product of the current density of the butterflies and net rate of growth of the butterfly population. The net rate of growth is the difference between the rate at which the population grows in the absence of wasps and the rate at which the wasps parasitize the butterflies. The second equation is the change in the wasp density over time. This is shown by the product of the current wasp density and the net rate of growth of the wasp population density. The net rate of growth is the difference between the rate at which wasps parasitize the butterfly larvae and the rate at which they die in the absence of butterflies.

We assumed that the rate at which the butterflies are parasitized and the rate at which the wasps parasitize the larvae depend on the concentration of aphrodisiac. We also assumed that the concentration released is inversely proportional to both the current wasp population density and the current butterfly populations density. It seems reasonable that the amount of anti-aphrodisiac released should decrease with a higher butterfly density and similarly should decrease with a higher wasp population density.

With our model, we assume that eventually, the system will balance out. Since both the wasps and butterflies depend on each other and the aphrodisiac, the system levels out to equilibrium. This is supported by the balance of nature theory as both the parasite and the host increase and decrease depending on the number of the other. We determined that as the system reaches high levels on one species, the other will have more of a chance to increase. For example, the more wasps in the area, the more aphrodisiac mixture will be needed in order to attract the wasps. This allows the females to release a low level of aphrodisiac to easily call a male without there being too much interaction with the wasps.