**Executive Summary**
**Problem C**

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**Background:**

*Pieris brassicae* is a species of large cabbage white butterflies, whose mating season is dependent on the release of various chemicals. Female butterflies often release chemical signals to attract males to their location. In response, male butterflies release anti-aphrodisiac to prevent other male butterflies from sensing the females chemical signals. Over time, parasitic wasps have developed the ability to detect anti-aphrodisiac. They have been using this ability to follow female butterflies and lay their own eggs in the butterflies eggs. This has lead to the butterfly population suffering from wasp larvae eating their eggs. Determine the long term interactions between the two species.

**Assumptions:**

We began by assuming a logistic growth model to model the future of the butterfly population. We chose this model because we assumed that the growth or decay of the butterfly population would be exponential. However, it was unrealistic to assume that the growth of the butterfly could be without limitations. Instead, we assumed a relative growth rate which approached a carrying capacity.

Furthermore, we assumed that the only factor affecting the growth rate of the population is the concentration of anti-aphrodisiac. We used the concept that as the concentration of anti-aphrodisiac increases, the number of eggs fertilized, as well as the number of eggs eaten, also increases. This lead to a formula for growth rate, R. R represents the net effect of the anti-aphrodisiac.

$$R = \frac{\#\ of\ eggs\ fertilized - \#\ eggs\ consumed}{total\ \#\ of\ eggs\ fertilized}$$

We assumed that the number of eggs consumed by the wasps is greater than the number of eggs fertilized. This causes R to be negative. This is assumed because we felt if the population were not suffering, then this would not be considered as a problem to the species.

Additionally, we assumed that the concentration of anti-aphrodisiac will begin by having a great effect on R. However, the concentration of anti-aphrodisiac will eventually reach a high enough level that its effect is negligible.

**Derivation:**

$$\frac{dP}{dt} = KP(1 - \frac{P}{K})$$

$$\int \frac{dP}{P(1-\frac{P}{K})} = \int k\ dt$$

$$P(t) = \frac{K}{1+Ae^{-Rt}}$$

Where,

$A = (K-P_0)/P_0$

t = time in millions years

$P_0$ = initial population in millions

K = Carrying Capacity

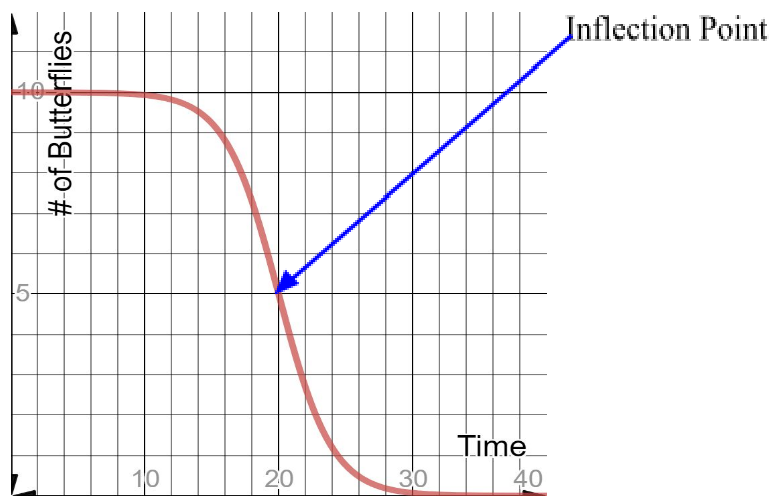
P(t) = future population in millions

**Results:**

We assumed current time to be the inflection point of the graph, and as a result, we shifted the graph horizontally to the right so that it would display the past, present, and future. The equation is shifted 20 million years for visual purposes. Based on this assumption, initial population occurs at the inflection point.

Assumed values:
K = 10 million butterflies
$P_0$ = 5 million butterflies
R= - 0.5
t = x in millions of years

$$P(t) = \frac{10}{1 + (\frac{10-5}{5})e^{-(-0.5)(t-20)}}$$




**Conclusion:**

The model predicts that the butterfly population will decay over time due to the wasps. The graph displays an inflection point, which occurs at:

$$\frac{d^2P}{dt^2} = 0$$

This represents the time when the rate of male butterflies fertilizing eggs equals the rate of the parasitic wasps eating the eggs.

$$\#\ of\ eggs\ fertilized = \#\ of\ eggs\ eaten$$

The model displays an outcome that does not account for the idea that species evolve and adapt to combat ecological problems. As a result, the butterfly population is displayed based on the assumption that it will go extinct after millions of years. Realistically, it is in the nature of all species to overcome a predator like the parasitic wasps.