A Model of Insect Chemical Espionage

SCUDEM IV 2019 - Problem C

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Introduction

We interpret, model, and analyze the interaction between the large cabbage white butterfly and parasitic wasps. In particular, we are interested in how selective pressure modulates the use of pheromones during the butterfly's reproductive process. We begin by providing a qualitative description of the interaction between butterflies and wasp, making a set of reasonable assumptions about the interaction, and proposing a model of the interaction dynamics. We then perform an analysis of our model and answer several selected questions regarding the behavior of the system.

Interaction Description

Female butterflies release pheromones that attract potential mates—an aphrodisiac. Males that detect the aphrodisiac race to locate the female. When a male mates with the female, it coats the female with an anti-aphrodisiac (AA) that deters competitors from approaching her. Now safe from the harassment of other males, the female butterfly can focus on finding an optimal location—leaves or plants on which hatched caterpillars will feed—to lay the fertilized eggs.

The adverse side effect of employing AA to facilitate reproduction is that parasitic wasps will also use the scent trail of the pheromone to locate female butterflies that have mated. When a wasp finds a female carrying fertilized eggs, it will latch on to the female's body until the butterfly lays its eggs. The wasp will then parasitize the butterfly eggs with its own eggs.

Note that the butterfly's survival depends on an adequate balance of AA males and non-AA males. If the AA males dominate, wasps will decimate the butterfly population. On the other hand, if non-AA males dominate, female butterflies will often be hindered from laying their eggs.

Assumptions

1. Wasps only attach to females with anti-aphrodisiac.
2. Male and female butterfly populations are equal.
3. All wasps are consistently prepared to lay eggs.
4. The population of wasps depends completely on the population of butterflies.
5. Every time male butterflies with anti-aphrodisiac interact with females, they successfully fertilize the eggs.
6. Wasps successfully parasitize butterfly eggs 50-100% of the time.
7. The butterfly population does not have a carrying capacity.
8. The female butterfly population never equals zero.

Model

$$\frac{dM_N}{dt} = aM_N F - b(M_N)^2 - cM_N M_A$$
$$\frac{dM_A}{dt} = eM_A F - fM_A FW$$
$$\frac{dW}{dt} = gM_A FW$$
$$\frac{dF}{dt} = \frac{dM_N}{dt} + \frac{dM_A}{dt}$$

Variables

M_N - male butterfly population not using AA

M_A - male butterfly population using AA

W -female parasitic wasp population

F -female butterfly population

Model Analysis

The non-AA male population increases when interacting with female butterflies. However, the more male butterflies, the less likely for successful fertilization because the females may continue to be bothered by male butterflies. The AA male population increases when interacting with females. The fertilized female butterflies are more likely to be parasitized by wasps because the wasps are attracted to the pheromones.

The wasp population increases when they interact with fertilized females with AA on them, assuming the wasp population depends completely on the butterfly population.

The female butterfly population is assumed to be equal to the male population. Using the sum rule of derivatives, it can be deduced that the change in the population of female butterflies is equal to the sum of the two male butterfly populations.

Selected Questions

How is this interaction balanced?

By limiting the use of AA, the butterflies increase their chance of surviving but also increase the stress on the female butterflies after mating. Although this limits the wasps' amount of butterfly nests that they can parasitize, they still can continue to live with the butterfly population as long as they limit how much their larvae eat.

What is the best balance for the system?

The best balance for the system will be for the males producing the AA to decrease their usage of it and therefore increase their likelihood of survival. Although the usage of the AA should be brought to a minimum, the males should still be encouraged to use the AA to be more successful in fertilization.

From the perspective of the wasps, the increase of the butterfly population is beneficial to them. Since the wasps are dependent on the mated female butterfly eggs they are more likely to procreate.

What is the long-term behavior of the system?

The system is characterized by an oscillation of the male butterfly populations. As one type of male becomes dominant the other will subside. This tendency will reverse when one population becomes too large and the other too small.

References

Martinus E. Huigens, Jozef B. Woelke, Foteini G. Pashalidou, T. Bukovinszky, Hans M. Smid, Nina E. Fatouros, Chemical espionage on species-specific butterfly anti-aphrodisiacs by hitchhiking Trichogramma wasps, Behavioral Ecology, Volume 21, Issue 3, May-June 2010, Pages 470–478, https://doi.org/10.1093/beheco/arq007