Differential equations for Modelling The population of Pieris brassicae and Trichogramma wasps

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Problem C

Problem

Female Pieris brassicae, the large white butterflies, emit anti-aphrodisiacs after mating to increase the chance of fertilization. However, the anti-aphrodisiac is a signal for Trichogramma brassicae and Trichogramma evanescens ,types parasitic wasp, to lay their eggs into the fertilized butterfly eggs.

Assumptions

- We combined both wasp groups into one group
- The ratio of female butterflies to the whole population is constant
- We used the average range of anti-aphrodisiacs, since we are dealing with larger populations of butterflies
- We assumed death rates, due to other causes than the parasitism, of butterflies and wasps

Approach and Process

We started with a basic predator prey model and added upper bounds (natural population bounds due to space, food, etc.)

$$\frac{\partial W}{\partial t} = (W_{max} - W)(\alpha W M - \beta W)$$

$$\frac{\partial M}{\partial t} = (M_{max} - M)(\gamma M - \delta WM)$$

W is the wasp population, M is the butterfly population, and
W_{max} and M_{max} representing the maximum population due to environmental restrictions.
The greek character variables are the ones that we altered to fit our specific wasp-butterfly model.

Now onto finding the relationship between the number of fertilized butterflies, the amount of anti-aphrodisiacs, and the chance of predation by the wasp. In the article “Chemical espionage on species-specific butterfly anti-aphrodisiacs by hitchhiking Trichogramma wasps” by Martinus and colleagues , they describe how anti-aphrodisiacs, attract wasps, at certain doses, and increase fertilization rates for butterflies.

So we described F, percent of fertilized females, and P, percentage of eggs taken by wasps, as functions of A, the amount of anti-aphrodisiacs.

f is the percentage of female butterflies in their population. We used 0.5 to represent a 1 to 1 ratio between male and female

The number of butterflies that are fertilized is
$fMF(A)$ where f is the ratio of female butterflies
The number of eggs that wasps take are
$fMF(A)P(A)$
The death rate of the wasps is represented by D_W
The death rate of the butterfly is represented by D_M
We can use these terms in our original equations to get our final model.

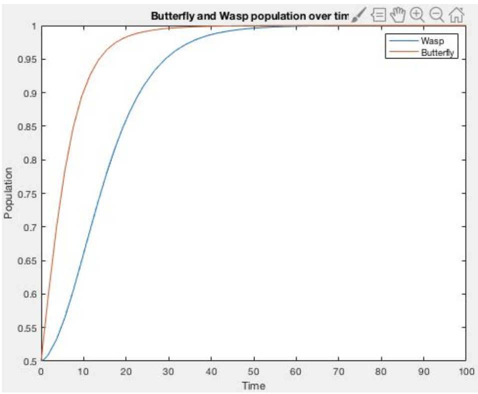
$$\frac{\partial W}{\partial t} = (1 - W)(-WD_W + WfMFP)$$
$$\frac{\partial M}{\partial t} = (1 - M)(fMF - WfMFP - MD_M)$$

Graphs

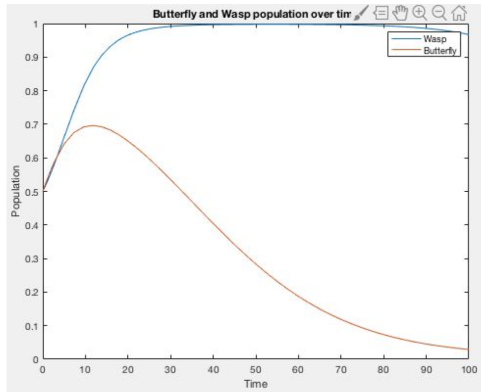
Constants:

F = 0.6 D_M = 0.1 D_W = 0.1 initial M and W = 0.5

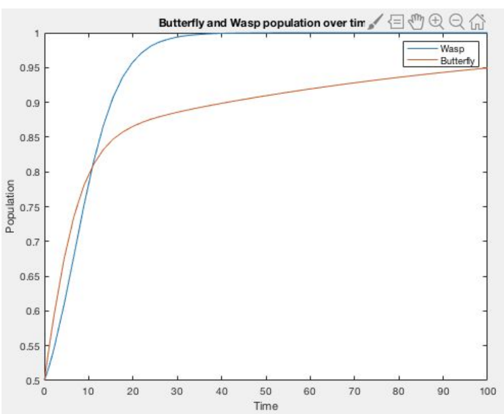
P = 0.9



P = 0.5



P = 0.75



Conclusion

With parameters we decided, the only times the wasp population and butterfly population did not decrease to 0 is when there was a high P value (0.75 and 0.9). This means that the chance of the wasps actually laying their eggs into a butterflies egg has to be very high to sustain its population given the circumstances we set. This does not seem to fit with what the article (Martinus) states. This may be an error in our initial settings Throughout our model building process we made assumptions to make our model simpler or easier to manipulate. When revising our model, we should consider these factors that we took out, such as the different types of wasps and how they behaved. This will hopefully help our model increase in accuracy at predicting real situations.

Works Cited

Martinus E. Huigens, et al. "Chemical espionage on species-specific butterfly anti-aphrodisiacs by hitchhiking Trichogramma wasps". *Behavioral Ecology*, Volume 21, Issue 3, May-June 2010, Pages 470–478, https://doi.org/10.1093/beheco/arq007