Problem C Morehead State University

By: Ryan Hinson Coach Name: Dr. Michael Dobranski

Russel Rogers

Suhana Ambol

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Statement of Problem: Design a model to determine the trade-offs and balance between male and female *P. brassicae* as well as the parasitic wasps.

The chemical interactions introduce two competing pressures on the butterfly population. For the male butterflies the anti-aphrodisiacs make it more likely for them to fertilize eggs. For the female butterflies the anti-aphrodisiacs make it less likely to be bothered by more males, and the females can focus on placing their eggs in the most advantageous place. On the other hand, the anti-aphrodisiacs make it more likely that the eggs will be eaten by the wasp larvae.

To develop our mathematical model, we take in consideration the population of male, female, and wasps at a particular time *t* as well as the reproduction between male and female butterflies at time *t*.

We have created four equations,

The current female population is made up of the new butterflies being born and the surviving female butterflies from the last year.

Here $df(t)$ is just some fraction of $current\ surviving\ butterflies\ f(t)$ and new born butterflies are a proportion of $R(t)\ i.e., kR(t)$.

$$\boldsymbol{F'(t) = dF(t) + kR(t)}$$

$$\boldsymbol{Where\ d, 0 \leq k \leq 1}$$

Similarly, the equation below represents rate of change of male population with respect to time t .

The only suggested change is $\boldsymbol{(1-k)R(t)}$ for the male proportion of reproduction.

$$\boldsymbol{M'(t) = dM(t) + (1-k)\,R(t)}$$

Wasps need to lay their eggs in the butterflies’ eggs in order to reproduce, the rate at which their population changes depend on the interaction between eggs and the wasps.

$$\boldsymbol{W'(t) = \theta R(t)W(t)}$$

$\boldsymbol{\theta}$ here represents the amount of anti-aphrodisiacs used by the female butterflies.

Finally, for the rate of change of reproduction with respect to time t

- Reproduction is directly dependent on the male and female interactions:

$$\boldsymbol{\alpha F(t)M(t)}$$

where **alpha** is some constant relating to the reproduction

- Reproduction is also partially dependent on how many eggs are killed by the wasps, which will be subtracted from the equation

$$-\boldsymbol{\delta W(t)}$$

where $\boldsymbol{\delta}$ is some proportion of the eggs that are killed or do not make it.

Therefore, the final product is:

$$\boldsymbol{R'(t) = \alpha M(t)F(t) - \delta W(t)} \qquad \textbf{where } \boldsymbol{\delta, \alpha \leq 1}$$

$$\textbf{Note: } \mathbf{W(t) \text{ and } R'(t) \geq 0}$$

<u>If we use more anti-aphrodisiacs</u>:

$$\boldsymbol{W'(t) = \theta R(t)W(t)}$$

We know $W'(t)$ will increase

$$\boldsymbol{R'(t) = \alpha M(t)F(t) - \delta W(t)}$$

$$\textbf{If, } \boldsymbol{\alpha M(t)F(t)=\delta W(t)}$$

Then,

$$\boldsymbol{R'(t)=0}$$

When, $\boldsymbol{\delta W(t) > \alpha M(t)F(t)}$ then the butterfly population fails to increase, but the wasp population increases due to the majority of the eggs being used to give birth to the wasps.

If this continues, the butterfly population will decrease significantly. When $M'(t)$ gets small enough then there will be no need for anti-aphrodisiacs

<u>If we use less anti-aphrodisiacs:</u>

$$\boldsymbol{W'(t) = \theta R(t)W(t)}$$

$$\boldsymbol{W'(t)} \textbf{ decreases}$$

$$\boldsymbol{R'(t) = \alpha M(t)F(t) - \delta W(t)}$$

$$\boldsymbol{R'(t)} \textbf{ increases}$$

We will see a rebounding of the butterfly population since there are more births not involving anti-aphrodisiacs.

If the rate at which it rebounds in the first couple years is smaller than the rate at which they are naturally dying off then there will be not be a “rebounding”. The population will ultimately die followed by the death of the wasps there after in which they decay at the same death rate.

Both the graphs will show **exponential decay**.

<u>Result</u>:

Population of butterflies will increase because reproduction is increasing. Over time as the population of males increases, there will be more of a need to use anti-aphrodisiacs for mating.

Two populations are not inversely related as the only way for butterflies to decrease is for them to naturally die and not reproduce as if they don't reproduce then there is no way for the wasps to reproduce either.

This then brings us to the situation where we increase anti-aphrodisiac use and begins a cycle where the two populations are not inversely related, but one will not increase unless the other is increasing.