Problem C: Chemical Espionage

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1. Summary

Some species of wasps behave as parasites to some species of butterflies. Insects like wasps and butterflies are also a food source for many birds. We wanted to study this host-parasite interaction between *P.rapae* butterflies and *T.Brassicae* as well as the predator-prey relationship between butterflies, wasps and birds to see how the population of the three species affect each other in the long run. Using differential Equations and Euler's method, we found that the best balance in the system is when bird to butterfly-wasp ratios are very high . We also found that wasps, butterflies and birds die in the long run.

2. Introduction

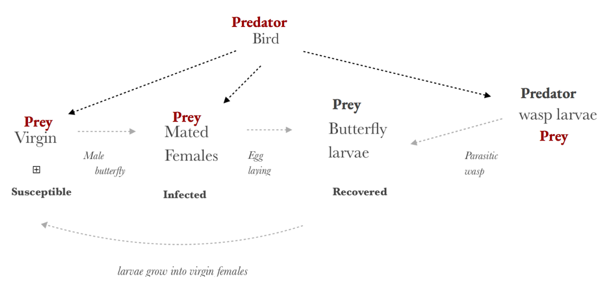
Female butterflies attract males using chemical signals. Because this signaling can attract many males, the males use other chemical signals to dissuade other males. However chemical signals also attract wasps who follow the butterfly and lay their own eggs in the butterflies' eggs. The wasps and butterflies also serve as a food source to birds. For this study we modeled the host-parasite relationship between this wasp and butterfly and predator prey relationship between the birds and insects to look at what the best balance for this system is and what is likely to happen to both populations in the long run.We modeled these group behaviors by using Euler's method. The software we used was MS Excel.

3. Assumptions

We assumed that the system is a 3-species environment with host (butterfly) and parasite+predator (wasp). Wasps were modeled to only attack butterfly larvae. Mated females to only reproduce once, after which they die. Our model also assumes no logistic growth. We also assume birds do not discriminate between virgin and mated butterflies. We also assumed that the population will grow naturally (birth and death exist).

4. The Model

We modeled the system based on SIRS and predator-prey systems.



Based on this model the system of equations that we have are-

$$dV/dt = -aVM + dLV - xBV$$
$$dM/dt = aVM - bLM - xBM$$
$$dL/dt = bLM - cLW - dLV$$
$$dW/dt = cWL - eW - yBW$$
$$dB/dt = xBV + xBM + yBW - zB$$

V(t)= number of *female virgin* butterflies at time *t*
M(t)= number of *mated female* butterflies at time *t*
L(t)= number of butterfly *larvae* at time *t*
Wt)= number of *wasps* at time *t*

$B(t)$= number of *birds* at time t

V_0= initial population at $t=0$ (female virgins)

M_0= initial population at $t=0$ (mated females)

L_0= initial population at $t=0$ (butterfly larvae)

W_0= initial population at $t=0$ (wasps)

B_0= initial population at $t=0$ (birds)

t= time (milli seconds)

a= fertilization rate constant for virgin females

b= death rate constant for mated females

c= susceptibility constant for attack against wasps by mated females

d= growth rate constant of butterfly larvae

e= natural death rate constant of wasps

x=rate at which bird eats butterflies

y= rate at which bird eats wasp

z= natural death rate of birds

5. Analysis/Results

Using Euler's Method on MS Excel we were able to see how the numbers of the population were changing over time. For this study we chose to look at the behavior with the following arbitrary conditions- $V_0 = 3000$, $M_0 = 200$,$L_0 = 2000$, $W_0 = 10$, t= 0.5 ms, a= 0.3, b= 0.9, c= 0.5, d= 0.02, e= 10. This gave us the graph in Figure-(a)

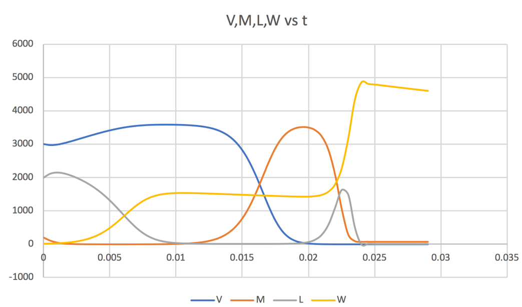


figure-(a)

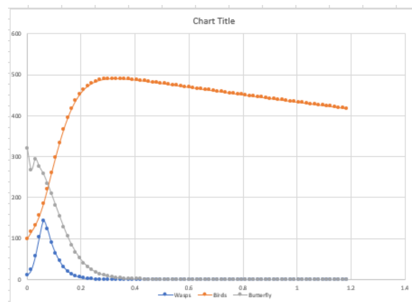


figure-(b)

Figure-(a) is the system in the absence f birds. As we can see, an abundance of virgin females in the butterfly population means corresponding population numbers of mated butterflies, M, is lower. Then, as numbers of mated butterflies increase (due to interaction with male butterflies), the numbers of virgin females go down. At the same time, increase in the number of mated females increases the number of butterfly larvae and in turn causes the wasp population to grow. Figure-(b) with initial conditions$V_0 = 300$, $M_0 = 20$,$L_0 = 200$, $W_0 = 10$, B_0=100, t= 0.5 ms, a= 0.3, b= 0.9, c= 0.5, d= 0.02, e= 10. is in the presence of birds and fit the model of a predator-prey system.

7. Conclusion

We concluded that for our model the best balance would be when the ratio of butterflies and wasps to birds is very high, perhaps 1:2. With higher numbers of wasps in the population, the butterfly populations begin to reduce rapidly. This also causes bird population to rise. In the long run, given our model, the number of butterflies in the system fall due to no births (since all mated females die after giving birth). This also causes a fall in the wasp population since their medium of birth- the butterflies- cease to exist. Hence both insect populations fall to zero. When this happens, bird population also starts to decrease due to no food source.