SCUDEM IV 2019

PROBLEM CHOSEN: C

CHEMICAL ESPIONAGE

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Presentation of Attempt

Mathematical modelling is the art of translating problems from an application area into tractable mathematical formulations whose theoretical and numerical analysis provides insight, answers, and guidance useful for the originating application.

Case Definition

We consider the mating behaviour of insects especially in Large Cabbage White Butterfly (*pieris brassicae*) which secrets a chemical signal(anti-aphrodisiacs) to dissuade other mates from its female partner. This has several implications including giving the female butterfly time for egg laying but also gives it the chance to be detected and followed to the spot of laying by parasitic wasps.

Problem Presentation

One big problem arising here is the death of the eggs as the chemical(anti-aphrodisiacs) makes it more likely these eggs will be eaten by the wasp larvae. This situation may lead to realistic decrease in population of the butterfly.

Considerations

- Develop a mathematical model for the interactions of the Male and female (white butterfly) and the parasitic wasp.
- Find the best balance for the system and what is likely to happen in the long run. Biology and Mathematics: Problem solving for mathematics has to do with numbers and various relationships that exist between them but this is usually not so in non-mathematics related fields.

However, this representation is needed in this said fields to help deduce logical reasoning and calculate variables discretely and arrive at a testable and reliable solution using mathematical formulas. Modeling therefore is the window.

Solution Approach

- We consider the White butterfly as a variable, say x and the Wasp as another variable, say y.
- We will use nonlinear differential equations to describe the dynamics of the biological systems in which this two species interact.
- Let x = number of white butterflies

  y = number of wasps

  t = time

Let $\alpha,\ \beta,\ \gamma,\ \delta$ be positive real parameters describing the interaction of the two species.

Relational Assumptions

• The wasp has constant need to follow the chemical signals

• The rate of change of population is proportional to its size

• During the process, the environment does not change in favour of one species, and genetic adaptation is inconsequential.

• The white butterfly may go into extinction.

Mathematical Representation

1. The butterflies are assumed to have unlimited amount of mating to reproduce exponentially unless subjected to predation. We represent this exponential growth by the term x.

2. We assume the rate at which the wasp detect the chemical to be proportional to the rate at which the wasp and the female mated butterfly meet represented by xy.

Solution Proceedings

• If $x = 0$ or $y = 0$, there would be no chemical detection.

• The rate of change of butterfly's population will then be given by its own growth rate minus the rate at which it is preyed upon.

Therefore

$$\frac{dx}{dt} = \alpha x - \beta x^* y \tag{1}$$

, where x^* = Number of butterfly preyed on

$$x^* \leq x$$

• The growth of the wasp is denoted by

$$\delta xy$$

• Note: The rate at which the wasp grows is not necessarily equal to the rate at which it becomes a parasite.

• The rate at which wasp is reduced due to either natural death or emigration is given by y which leads to an exponential decay in the absence of the wasp.

• Therefore, the rate of change of the wasps population depends on the rate at which it is a parasite on the White butterfly minus its intrinsic death rate.

$$\frac{dy}{dt} = \delta xy - \gamma y \tag{2}$$

We shall now adopt the Lotka-Voltera equations to solve the two generated nonlinear differential equations for the dynamics of the biological systems in which this two species interact.

- Given by

$$\frac{dx}{dt} = \alpha x - \beta \epsilon x y \quad \text{and} \quad \text{let } x^* \;=\; \epsilon x \tag{3}$$

Let both (3) and (2) = 0 using the steady-state solution
we have,

$$y \;=\; \frac{\alpha x}{\beta \epsilon x}$$

sub this into (2), we obtain

$$y \;=\; \frac{\alpha}{\beta \epsilon}$$

then

$$x \;=\; \frac{\epsilon}{\delta}$$