Problem C Executive Summary
Oxford College of Emory University

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Statement of the Problem C

Determine the trade-offs and balance for *Pieris brassicae* through the usage of anti-aphrodisiacs (*a.a.)*

Model Design

We simulate an idealized ecosystem with only two species involved, the white butterfly *P. brassicae* and the egg parasitoid wasps from the genus *Trichogramma*. Male butterflies may encounter other males during reproduction. To either mask or dissuade other males, male butterfly transfers a chemical signal, a.a., to the mated female. Thus, such interactions between female and male butterflies increase the probability for females to fertilize eggs. On the other hand, the *a.a.* that mated females carry attracts parasitic wasps, making the newly laid eggs more likely to be eaten. This model analyzes the dynamic trade-offs affecting the survival of eggs through the usage of *a.a.*

Assumptions

1. This is an idealized ecosystem focusing solely on wasps and butterflies. The mortality rate for butterflies and the birth rate for wasps are of this model's interest. We assume that the birth rate for butterflies and the mortality rate for wasps are consistent with natural conditions.
2. Even though *T. brassicae* needs a successful ride to develop *T. evanescens'* innate ability, other aspects of predatory strategies are practically similar for the two wasp species (Huigens et al., 2010). Thus, we combine them.
3. The ratio of female *P. brassicae* to male *P. brassicae* is 1:1, and the probability of giving birth to a male progeny is 50%.

Variables

t: time (days)
B, W: the total population of butterflies *P. brassicae* and wasps *T. brassicae,* respectively
g_B: the growth rate of butterflies with the presence of *a.a.*
f_1: butterflies' mortality factor caused by supplying eggs to wasp with the presence of *a.a.*
f_2: wasps' growth factor caused by consuming host butterfly eggs with the presence of *a.a.*
α: the amount of *a.a.* secreted by one male butterfly (μg)

Constants

K_B , K_W: the carrying capacity of butterflies and wasps in the idealized ecosystem, respectively
d_W: the natural mortality rate of wasps.
g_{B0}: the natural growth rate of butterflies with the absence of *a.a.*
f_{10}: butterflies' natural mortality factor with the absence of *a.a.*
f_{20}: wasps' natural growth factor with the absence of *a.a.*
m: the effectiveness of *a.a.* in helping female butterflies to better reproduce eggs
h: the effectiveness of *a.a.* in helping wasps eat the eggs

Models

$$\frac{dB}{dt} = g_B B\left(1 - \frac{B}{K_B}\right) - f_1 BW \qquad (1)$$

$$\frac{dW}{dt} = -d_W W\left(1 + \frac{W}{K_W}\right) + f_2 BW \qquad (2)$$

where $g_B,\ f_1, and\ f_2$ can be obtained from following system of equations:

$$g_B = \ g_{B0}(1 + m\alpha) \qquad (3)$$

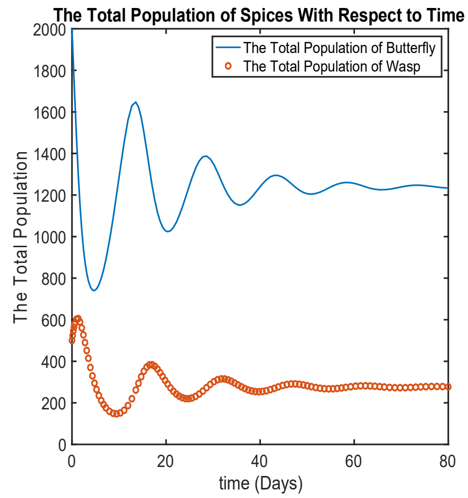
$$f_1 = f_{10}(1 + h\alpha) \qquad (4)$$

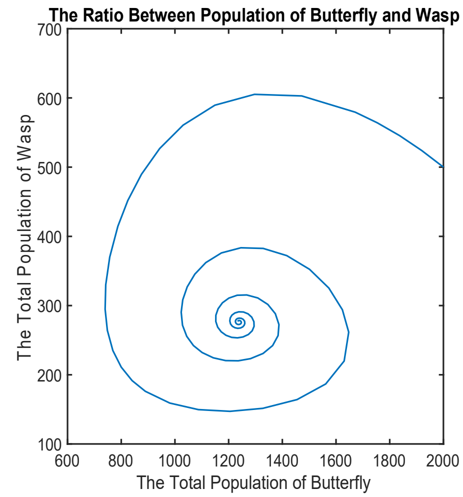
$$f_2 = f_{20}(1 + h\alpha) \qquad (5)$$

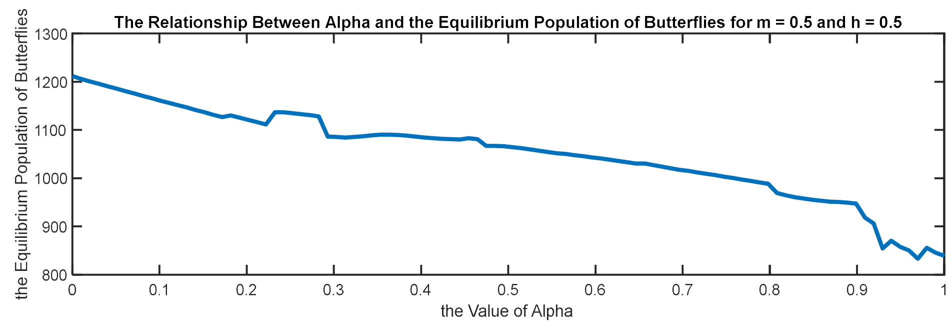
Model Explanation

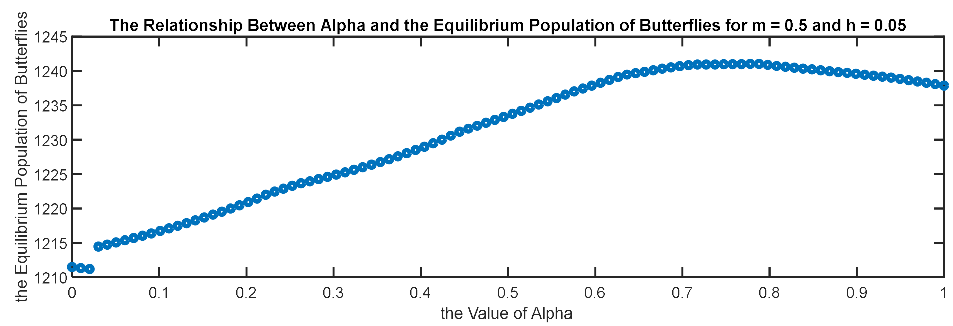
The established Lotka-Volterra Model fits well into our simulated situation of the competition between butterflies and wasps. To modify the classic LV model, we introduced the influence of a.a. on the butterflies' mortality factor due to interactions with wasps (f_1), wasps' growth factor due to interactions with butterflies (f_2), and the natural growth rate of butterflies (g_B). We suppose that the level of secretion of a.a. has a linear relationship with each of them. Thus, we set up a linear relationship for f_1 and f_2 with new variables: h and α. Also, we make a similar equation for g_B with new variables: m and α. Our goal is to find the best level of α that maximizes the butterfly population without changing other constants. Initial conditions are 2000 butterflies and 500 wasps at t = 0. Other constants include m = 0.5, h = 0.5 (Fatouros et al., 2005), g_{B0}= 0.348, d_W = 0.57(Southon et al., 2015), K_B = 6000, K_W = 3000, f_{10} = 0.01, and f_{20} = 0.0005.

Simulation and Analysis









Our simulation has several characteristics of LV models. Initially, with many wasps endangering the butterflies, the butterfly population reaches its minimum around day 5. Then, as the wasp population remains at a high level, the butterfly population starts to recover. With increasing number of butterflies, the wasps can stage a comeback, forming a cyclical repeat. As time increases, these curves will have low amplitude sinusoidal shape, and both the butterfly population and the wasp population will vary slightly with time. We call it equilibrium population or fixed point.

Under our assumptions m = 0.5 and h = 0.5, we found that α and B have a negative relationship. Thus, when α = 0, the equilibrium population of butterfly has a maximum value of 1212. The disadvantage of releasing a.a. always outweigh its advantage. However, if we decrease h, for example, to 0.05, we find a quasi-parabolic curve for α and B. The butterfly population is maximized at 1241 with α = 0.78. Male butterflies have an incentive to release a.a. until α reaches 0.78. Studies show that the male butterflies are under selective pressure and tend to minimize the use of a.a., which is consistent with our simulations.