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Executive Summary: Chemical Espionage

Introduction:

One of many ways which insects compete for the ideal mate is through the process of releasing chemical compounds, called pheromones. Each pheromone produces a signal and a certain response. For species such as the large cabbage white butterfly (*Pieris brassicae*), the anti-aphrodisiacs released by male butterflies during copulation is exploited by parasitic wasps (*Trichogramma brassicae*). The wasps will then hitch a ride on the fertilized female butterfly to the oviposition of the eggs and parasitize the batch. The anti-aphrodisiac creates two competing pressures on the butterfly population. 1. Males can fertilize eggs and females will be unbothered by other males to focus on oviposition. 2. It is more likely eggs will be eaten by wasp larvae.

Statement of Problem:

We are asked to develop a mathematical model for the interactions of the male and female *P. brassicae* as well as the parasitic wasps with the use of anti-aphrodisiac.

Assumptions:

1. Eggs are the only resource for the parasitic wasp.
2. Population of males does not have an appreciable impact on the number of fertile female butterflies.
3. Logistic growth for butterflies.
4. The eggs parasitized is proportional to the number of wasps.
5. Continuous rate of new butterflies and eggs being made year-round.

Model:

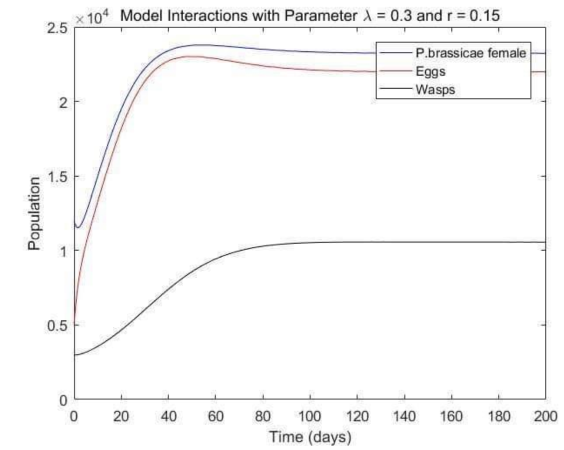
1. $\frac{dB}{dt} = \lambda E(1 - \frac{B}{K}) - k_B B$
2. $\frac{dE}{dt} = \sigma B - \lambda E - r\frac{EW}{E+W}$
3. $\frac{dW}{dt} = jr\frac{EW}{E+W} - k_W W$

- $B = number\ of\ female\ butterflies$
- $E = butterfly\ eggs$
- $W = wasps$
- $j = wasps\ larvae\ scaling\ factor$
- $K = carrying\ capacity\ of\ female\ butterflies$
- $\lambda = percent\ of\ eggs\ hatched\ into\ female\ butterflies$
- $\sigma = fertility\ rate\ of\ female\ butterflies$
- $r = rate\ of\ parasitized\ eggs$
- $k_B = death\ rate\ of\ female\ butterflies$

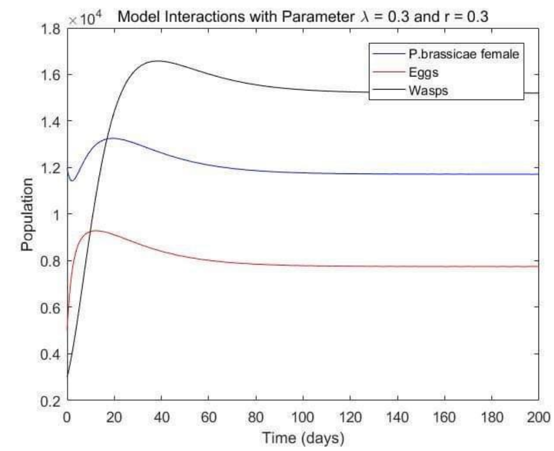
Results and Discussion:

Based on our model, the population of the butterflies can be driven to extinction if the stealing rate of the parasitized eggs is increased past a certain threshold. The threshold is related to the growth rate of the butterfly. In Graphs 1-4 below, we fix λ and incrementally increase r to provide a qualitative understanding of the mechanics. Graphs 1, 2, and 3 result in stable population levels for the butterflies and the wasps. Graph 4 demonstrates the population of the butterflies being driven to extinction. Graph 3 provides data which seems to indicate the butterfly population is being driven extinct but stabilizes for small values of each species. However, in Graph 4, the populations become extinct. According to the problem, both the natural growth rate of the butterflies and the parasitizing rate of the wasps increase with the use of the anti-aphrodisiac by the butterflies. Hence, to determine the long-term behavior of our system one would need to conduct an experiment to verify if the parasitizing rate grows faster than the natural growth rate of the butterflies under this influence. If the parasitizing rate does grow faster than the natural growth rate of the butterflies, then increasing the use of the anti-aphrodisiac should result in population decline as seen in Graphs 3-4. Under this condition, the butterflies would be under strong selective pressure to reduce their use of the anti-aphrodisiac when mating.

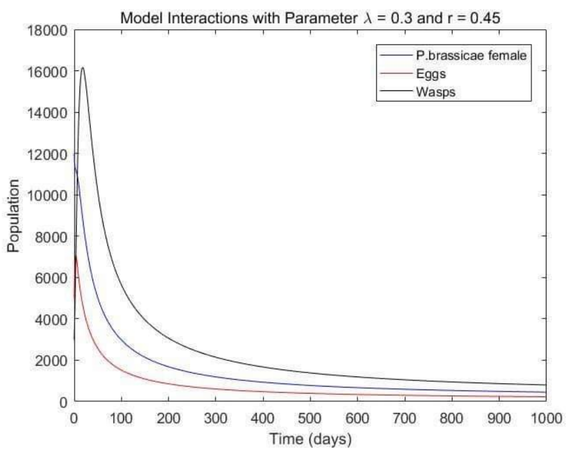
Graph 1



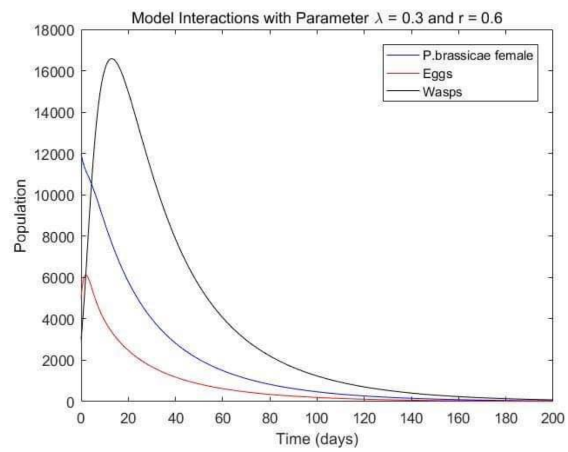
Graph 2



Graph 3



Graph 4



Limitations:

Our model does not consider additional resources for the wasps. It also does not consider the presence of additional predators which might prey on the wasps or the butterflies. Additionally, it attempts to model a discrete system with continuous functions, which leads to inherent rounding errors and inconsistencies at low values of our discrete variables.

Conclusion:

If the parasitic wasps steal enough butterfly eggs to overwhelm the butterfly's natural growth rate, the butterflies will be driven to extinction. The egg stealing capabilities of the parasitic wasps are directly related to the use of the anti-aphrodisiac but using anti-aphrodisiac can also drive up the natural growth rate of the butterflies. However, there are also stable levels of parasitic activity which will allow both populations to flourish.