Problem C Executive Summary

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Problem C asks us to model the interactions between certain species of butterflies (*Pieris brassicae*) and parasitic wasps to determine an appropriate balance between butterflies mating and attracting parasitic wasps. The main complication in this interaction is the release of anti-aphrodisiac chemicals by male butterflies to promote monogamy in the species. While this chemical release discourages male butterflies from mating with a fertilized female, it also attracts the parasitic *Trichogramma* wasps. Our model will attempt to show how the male and female members of each species interact within and across species to determine a balance in the system and what is likely to happen in the long term.

To simplify species interactions, we assume the two species of wasps are functionally the same, and treat them as one. This is due to their similar interactions with the butterflies. To create a usable model, we make several assumptions about the system. First, we assume that both populations are exactly half male and half female, as shown.

$$B_m(t) = B_{fem}(t) = \frac{1}{2}B(t)$$

$$W_m(t) = W_{fem}(t) = \frac{1}{2}W(t)$$

We also assume that fertilized females of both species are not eligible for mating, while unfertilized females and all males are always eligible. Additionally, the environment has limited resources, so carrying capacities (K_B and K_W) are applied to both species. The geometric average of male (B_m) and unfertilized female (B_{fem}-B_{fert}) butterflies is used to make the model behave in such a way that if there are no males or no unfertilized females, mating will occur. Females are unfertilized when they lay eggs, which happens at a designated time (T_B and T_W) after mating. The rate of butterfly fertilization is proportional to the amount of anti-aphrodisiac released (A).

$$\frac{d}{dt}B_{fert}(t) = c_1 A \frac{K_b - B(t)}{K_b}\sqrt{\left(B_{fem}(t) - B_{fert}(t)\right) B_m(t)} - c_1 A \frac{K_b - B(t-T_b)}{K_b}\sqrt{\left(B_{fem}(t-T_b) - B_{fert}(t-T_b)\right) B_m(t-T_b)}$$

$$\frac{d}{dt}W_{fert}(t) = c_2 \frac{K_w - W(t)}{K_w}\sqrt{\left(W_{fem}(t) - W_{fert}(t)\right) W_m(t)} - c_2 \frac{K_w - W(t-T_w)}{K_w}\sqrt{\left(W_{fem}(t-T_w) - W_{fert}(t-T_w)\right) W_m(t-T_w)}$$

A wasp laying eggs in a butterfly nest will be referred to as “hijacking”. We assume that butterflies laying eggs, wasps finding butterfly nests, and wasps laying eggs all happen at the same time. The number of hijackings is found with a harmonic average of the butterflies and wasps laying eggs, with the wasps weighted by the chemical release.

$$H(t) = c_3 \frac{A B_{fert}(t-T_b)\, W_{fert}(t-T_w)}{A\, B_{fert}(t-T_b) + W_{fert}(t-T_w)}$$

The rate of butterfly eggs laid is proportional to the number of butterflies laying eggs whose nests are not hijacked. If a wasp successfully lays larvae in a butterfly’s egg nest, no butterflies from that nest survive. The rate of wasp eggs laid is proportional to the rate of hijacked nests. The proportions are the number of eggs each species lays, which is assumed to be constant. Eggs hatch after a constant amount of time (M_B and M_W).

$$\frac{d}{dt}E_b(t) = L_b\left(\frac{d}{dt}B_{fert}(t) - H(t)\right) - L_b\left(\frac{d}{dt}B_{fert}(t - M_b) - H(t - M_b)\right)$$

$$\frac{d}{dt}E_w(t) = L_w H(t) - L_b - H(t - M_w)$$

Members of each species are eligible to mate immediately after maturing, so we say the change in the population is the species' birth rate minus the death rate. The birth rate is equal to the rate of eggs laid one maturing period ago, and the death rate is equal to the birthrate one lifetime ago.

$$\frac{d}{dt}B(t) = \frac{d}{dt}E_b(t - H_b) - \frac{d}{dt}E_b(t - H_b - \mathrm{D}_b)$$

$$\frac{d}{dt}W(t) = \frac{d}{dt}E_w(t - H_w) - \frac{d}{dt}E_w(t - H_w - \mathrm{D}_w)$$

Ideally, we would use collected data to curve fit our model using the constants while also checking to see if our choices in modeling the populations were correct. Since this system is non-linear, we numerically approximated a solution using Euler's method.

We tested our model with values of A on [0,1], with 0 bein no anti-aphrodisiac released, and 1 being the maximum release. A value of 1 maximizes the steady-state population of butterflies. Since there is no scale related to the model due to a lack of data, a higher value of A simply corresponds with a greater amount of anti-aphrodisiacs being released. This is the biggest issue with our model, since the information given by the problem and its reference tells us that the butterflies are under pressure to use as little anti-aphrodisiacs as possible.

The main reason for why the model finds a greater release to be more beneficial for the butterflies is due to the wasps' failure to generate a population size comparable to the butterflies. This means that at no point do they pose a serious threat to the butterfly population.