Solution to Chemical Espionage Problem

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I. Introduction

Due to the special relationship between the butterflies and wasps, and nonlinear interaction for male and female butterflies, we considered sophisticatedly for the specific relationship of male, female butterflies, and wasps. Our team modeled the relationship between Pieris Brassicae and wasps using several different factors.

For solving the problem, the very first thing that came to our mind was the population model. This model fits this problem well because it provides all the factors that we should take into consideration: birth and death rate, impact of wasps, and communication between male and female butterflies. Beyond that, we believe that anti-aphrodisiac is the key to the problem because butterflies use it to give birth, while wasps also locate eggs with the help of this chemical.

II. Analysis

1. Assumption

1) Every time each female interacts with a male, a certain amount of anti-aphrodisiacs will be released by the male.

2) The number of males and females butterflies are about the same. Therefore, the number of male/female butterflies is half of the whole population.

3) When the system reaches its best balance, it contains similar numbers of butterflies and wasps.

4) The interaction of butterflies and wasp can be described with predator-prey equation, where we assume wasps depend on butterflies as food resources to survive. In addition, the growth of the butterfly population to be logistic.

2. Notation

Female Butterfly Population at t years	$\mathrm{F}(t)$
Male Butterfly Population at t years	$\mathrm{M}(t)$
Butterfly Population at t years	$B(t) = \mathrm{F}(t) + \mathrm{M}(t)$
Wasp Population at t years	$\mathrm{W}(t)$
Butterfly Birth Rate	$\alpha + \beta B(t) - \gamma B(t)W(t)$
Butterfly Death Rate	$x + yB(t)$
Wasp Birth Rate	a
Wasp Death Rate	$b - cB(t)W(t)$

III. Model

Population model of Butterfly Population by time:

$$\frac{dB}{dt} = \left(\alpha + \beta F(t) - \gamma F(t)W(t)\right)(F(t) + M(t)) - \Big(x + y\big(F(t) + M(t)\big)\Big)\left(F(t) + M(t)\right)$$

$$\frac{dB}{dt} = \left(\alpha + \beta B(t) - \gamma B(t)W(t)\right)B(t) - \left(x + yB(t)\right)B(t)$$

Population model of Wasps Population by time:

$$\frac{dW}{dt} = aW(t) - \left(b - cF(t)W(t)\right)W(t)$$

$$\frac{dW}{dt} = aW(t) - \left(b - cB(t)W(t)\right)W(t)$$

α is the natural birth rate of butterflies. $\beta F(t)$ shows the effect of anti-aphrodisiac produce by interaction between male and female butterflies on their birth rate. $\gamma B(t)W(t)$ shows the effect of interaction between butterflies and wasps on butterflies' birth rate. $x + yB(t)$ shows the natural death rate and death rate depends on capacity for butterflies.

a is the natural birth rate of wasps. b shows the natural death rate of wasps. The interaction between wasps and butterflies provide food for wasps' larva due to the effect of anti-aphrodisiac. $cB(t)W(t)$ shows the effect of this interaction on wasps' death rate.

IV. Simulation

1. Simulation Setup

We set up the differential equations with a range of possible values for the constants and calculate the population of wasps and butterflies in the long run. We intend to find out reasonable constant values so that the ecosystem can eventually reach a balance. Also, we modify each constant to find out its individual effect on the insects populations.

2. Simulation Assumptions

1) We assume that butterflies as prey has a relatively high birth rate and low death rate.

2) We assume the natural birth rate of wasps is lower that the natural death rate as they depend on butterflies to survive.

3. Simulation Result

When

$\alpha = 0.2\,, \beta = 0.9\,, \gamma = 0.3\,, x = 0.001\,, y = 0.01$

$a = 0.01\,, b = 0.05\,, c = 0.005$

the system reaches its best balance where the populations of both wasps and butterflies reaches around 3.15 (thousands) in the long term.

V. Result

1) In the long run, both the populations of butterflies and wasps intend to be a constant, which indicates that the system reaches an equilibrium.
2) The populations of butterflies and wasps in the long run tend to remain unchanged regardless of the initial conditions, as the constants remain unchanged.
3) The positive effect of anti-aphrodisiac on butterflies birth rate indeed causes the population of butterflies in long run to decrease while causes the population of wasps to increase.
4) When the interactions among butterflies and wasps causes less birth rate of the butterflies, butterflies population in long run increases while wasps population in long run decrease.